MATHEMATICS 30

LEARNING FACILITATOR'S MANUAL

Distance Learning

Alberta EDUCATION

Mathematics 30

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Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Contents

Contents

Teacher's Copy of the Final Test with Marking Guide

Final Test

Teacher Questionnaire

Introduction

A survey of these course materials will confirm that this new learning package has been specially designed for many kinds of teachers working in a variety of situations.

Which Category Do You Fit?

☐ Small Schools Teacher

    ☐ inexperienced

    ☐ experienced, but in other subject areas

    ☐ experienced in teaching Mathematics 30 but wanting to try a different approach

☐ Distance Learning Teacher

    ☐ travelling to schools within the jurisdiction

    ☐ using facsimile and teleconferencing to teach students within the area

☐ Larger Schools Teacher

    ☐ inexperienced

    ☐ experienced in teaching Mathematics 30 but wanting to try a different approach
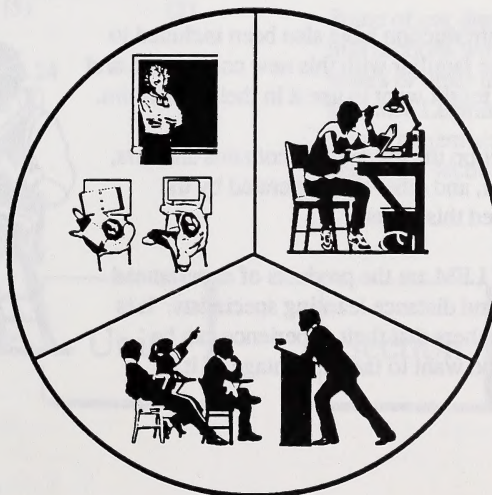
Because these materials have been created by experienced classroom teachers and distance learning specialists, they have many advantages for students and teachers regardless of their situation.

Advantages for Students

- incorporates a strong learner-centred philosophy

- promotes such qualities in the learner as autonomy, independence, and flexibility

- is developed through media which suits the needs and circumstances of the learner

- reflects the experiential background of Alberta students

- opens up opportunities by overcoming barriers that result from geographical location

- promotes individualized learning, allowing learners to work at their own pace

Advantages for Teachers

- allows teachers maximum teaching time and minimizes preparation time

- includes different routes through the materials to suit different learners

- incorporates a wide range of teaching strategies, in particular those using independent and individual learning

- delivers curriculum designed by education specialists that reflects Alberta Education Program of Studies with an emphasis on Canadian content

- provides learning materials which are upwardly compatible with advanced educational technology

Does it sound like something you could use?

The student materials are not the only components designed for independent, guided instruction; so is this Learning Facilitator's Manual. It begins with an overview of the current Alberta Education Program of Studies for high school mathematics. This summary is included for inexperienced teachers or those teachers who have found themselves teaching high school mathematics when their training is in other subject areas. This brief summary is not meant to replace the Alberta Education Program of Studies, but rather to help teachers confirm the highlights of the program.

Other parts of this introduction have also been included to help teachers become familiar with this new courseware and determine how they might want to use it in their classroom.

Beyond the introduction the guide itself contains answers, models, explanations, and other tips generated by the teachers who authored this course.

The courseware and LFM are the products of experienced classroom teachers and distance learning specialists. It is the hope of these teachers that their experience can be shared with those who want to take advantage of it.
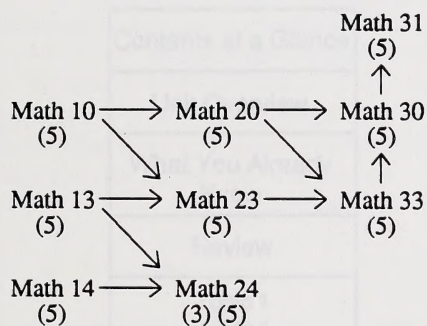
Overview of the Program of Studies

A knowledge of mathematics is essential in many other areas of learning. With available information and the use of technology the emphasis will not be so much on memorizing formulas but on how to use mathematical information to solve problems. The student will be expected to construct, investigate, translate, discuss, analyze, and formulate mathematics in a dynamic way.

The Mathematics 10 – 20 – 30 – 31 sequence is designed for students with an interest and aptitude in mathematics who are intending to pursue postsecondary studies at a university or in a mathematics-intensive program at a technical school or college. Having successfully completed Mathematics 30, students will have fulfilled the mathematics requirement for the Advanced High School Diploma.

The Mathematics 13 – 23 – 33 sequence is the mainstream mathematics program designed for students who require mathematics to prepare for universities, colleges, trades, and employment. The Mathematics 13 – 23 – 33 sequence will satisfy the requirements for the General High School Diploma.

The Mathematics 14 – 24 program is a general series suitable for students who have experienced difficulties in previous mathematics courses.

The complete high school mathematics program is illustrated below along with recommended transfer points. The number in brackets indicates credits.

```
                                    Math 31
                                     (5)
                                      ↑
Math 10 ──→ Math 20 ──→ Math 30
  (5)    �‿    (5)    ↘    (5)
            ↘       ↘      ↑
Math 13 ──→ Math 23 ──→ Math 33
  (5)    ↘    (5)         (5)
            ↘
Math 14 ──→ Math 24
  (5)       (3) (5)
```

This mathematics course is divided into units. Each unit covers a major area of mathematical study, and is divided further into specific content areas called topics. Each unit has its own assignment booklet which the student completes and submits at the end of that unit.

Some of our distance learning courses are divided into sections within modules. If you receive letters or other materials from the Alberta Distance Learning Centre which refer to modules, please remember that a module is the same as a unit and a section is the same as a topic.

Module	=	Unit		Section	=	Topic
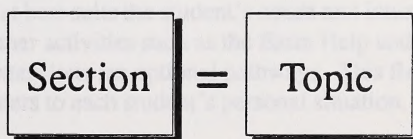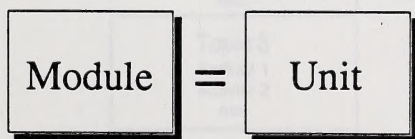
Overview of Mathematics 30

Unit 1: Polynomial Functions

Unit 2: Logarithms

Unit 3: Sequences, Series, and Limits

Unit 4: Trigonometry

Unit 5: Quadratic Relations

Unit 6: Statistics

Unit 7: Permutations and Combinations

The Mathematics 30 course is the final course in the Mathematics 10 – 20 – 30 academic series. The course is divided into seven units as outlined above. The course is offered for five credits only.
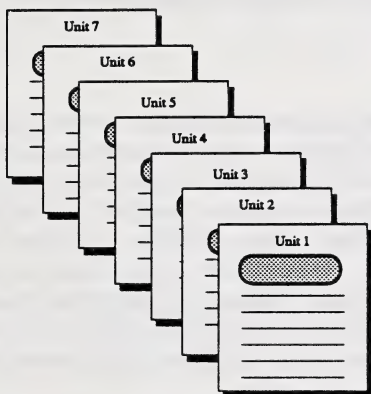
Structure of the Learning Package

Basic Design

This new learning package involves many other components in addition to the Learning Facilitator's Manual.

Units

The print components involve many booklets called units. These units contain guided activities that instruct students in a relevant, realistic setting.

The units have been specially designed to promote such qualities in the learner as autonomy, independence, and flexibility. Writers have incorporated such teaching strategies as working from the concrete to the abstract, linking the old to the new, getting students actively involved, and using advance, intermediate, and post organizers. Many other techniques enable learners to learn on their own for at least some of the time.

The structure of the unit booklets follows a systematic design. Each unit begins with a detailed table of contents which shows the students all the main steps. It acts as an organizer for students. The unit overview introduces the unit topic or theme. A graphic representation has been included to help visual learners and poor readers. The course overview also states the weightings of each assignment.

The body of the unit is made up of two or more closely related topics. Each topic contains student activities that develop skills and knowledge centred around an objective.

The activities may involve print, audio, video, and computer formats. At times the student and the learning facilitator are allowed to choose the activity that best suits the student's needs and interests. Other activities such as the Extra Help and Extensions are optional pathways. This flexibility caters to each student's personal situation.

The summary focuses on the skills and strategies that the student has learned.

Contents at a Glance
Unit Overview
What You Already Know
Review
Topic 1 Activity 1 Activity 2 etc.
Topic 2 Activity 1 Activity 2 etc.
Topic 3 Activity 1 Activity 2 etc.
Unit Summary
Appendices

Assignment Booklet
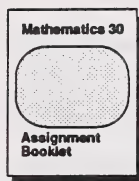
Accompanying each unit is an assignment booklet. The activities in these booklets can be used for formative and for summative assessments. The students should complete these assignment booklets when they have thoroughly reviewed the unit materials. The assignment booklets have been designed for classroom use, for faxing, or for mailing. **If the booklets are not being mailed, you should remove the outside cover.**
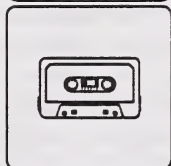
Media

COMPUTER DISK

VIDEOCASSETTE

AUDIOCASSETTE

The package also includes reference to media. Pathways have been developed so students can use a variety of media to achieve the objective. These different routes have been included to suit different learners. Wherever videos or computer software have been included, a print pathway is also available. This way, if the media resource isn't available or desired, a student can follow the print pathway and still successfully achieve the objective.

Special audiocassettes guide the student through a difficult concept. The appearance of the audiocassette icon reminds students that there is this additional help available.

If the students are working individually, you may find this cassette a valuable asset. If you are working in a large group, you may wish to guide the students yourself.

Materials, Media, and Equipment

Mandatory Components

Equipment (Hardware)	Media	Materials
• audiocassette player	• prepared audiocassettes (come with learning package)	• LFM for Mathematics 30 • one complete set of unit booklets (7) and assignment booklets (7) for each student • geometry set • scientific calculator or graphics calculator • There is a final test.

Optional Components

Equipment (Hardware)	Media	Materials
• VCR	• videocassettes Videocassettes used in the course may be available from the Learning Resources Distributing Centre or ACCESS Network. You may also wish to call your regional library service for more information.	

Using This Learning Package in the Classroom

Conventional Classroom

Whether your classroom has desks in rows or tables in small groups, you may be most comfortable with a learning system that you can use with all your students in a paced style. In other words, you may want a package that will suit all of your students, so they can move through the materials as one group or several small groups. Because these materials contain different routes or pathways within each unit, they can address various learning styles and preferences. The materials also include many choices within the activities to cater to different thinking levels and ability levels. Because of their versatility and flexibility, these materials can easily suit a conventional classroom.
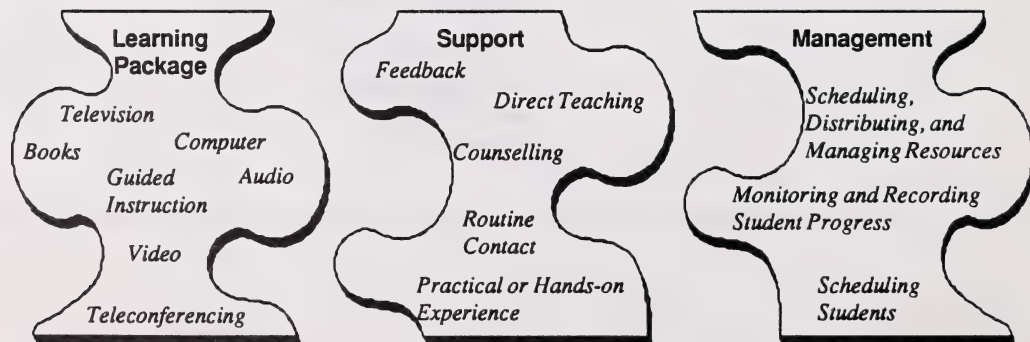
Open-Learning Classroom

Open-learning is the concept of opening up opportunities by overcoming barriers of time, pace, and place by giving the learners a package specially designed to enable them to learn on their own for at least some of the time.

Such a concept is not new. Many teachers can recite attempts to establish an individualized learning system as they recognized the importance of trying to personalize courseware to meet each individual student's needs. But these efforts often failed, due to lack of time and lack of quality materials that conformed to Alberta specifications.

Due to advanced educational technology and improved Alberta-specific learning packages, a student-centred approach is now possible. Improved technology now allows us to provide support to learners, individually regardless of their pace or location. A teacher cannot be in twenty-eight places at one time offering guidance. However, media and a well-designed learning package can satisfy individual needs. Technology can also help provide an effective management system needed to track the students as they progress independently through the materials.

The key to a successful open-learning system depends on three vital elements: a learning package specially designed to enable students to learn effectively on their own for at least some of the time; various kinds of learner support; and a management system and style that ensures that the open-learning system runs smoothly.

The Key to a Successful Open-Learning System

Learning Package
Television
Books
Computer
Guided Audio
Instruction
Video
Teleconferencing

Support
Feedback
Direct Teaching
Counselling
Routine Contact
Practical or Hands-on Experience

Management
Scheduling, Distributing, and Managing Resources
Monitoring and Recording Student Progress
Scheduling Students

Learning Package

The specially designed learning package needed for a successful open-learning system has been developed for you. The objectives teach current Alberta specifications using strategies designed for individualized instruction. As the learning facilitator, you need to be sure to have all the components in the learning package available to students as needed.

If adequate numbers of media are available to satisfy the demand, a centre can be established for specific media.
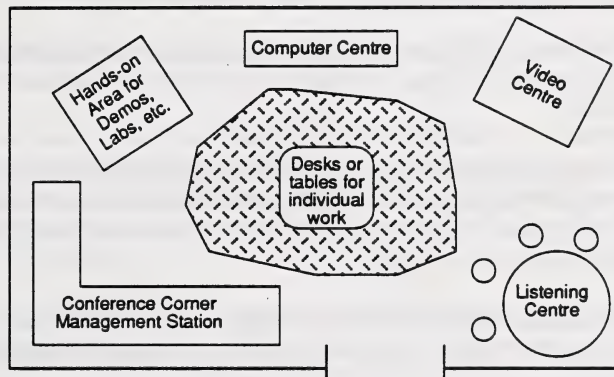
You may not have the luxury to have enough hardware to set up a permanent video or computer centre in your classroom. In that case, students should be encouraged to plan ahead. Perhaps every three to five days they should preview their materials and project when they would need a certain piece of media. This would allow you to group students, if necessary, or reserve media as required.

Support

Support is definitely a key element for successful learning, and when you're planning an individualized, non-paced program, you need to carefully plan when and how support will be given.

The materials contain a form of consistent support by providing immediate feedback for activities included in the unit booklet. High school students have solutions, models, explanations, and guides included in the appendix of every unit booklet. These are included so students can receive immediate feedback to clarify and reinforce their basic understanding before they move on to higher levels of thinking.

As the learning facilitator, you may be needed to offer more personal guidance to those students having difficulty, or you may need to reinforce the need for students to do these activities carefully before attempting the assignments in the assignment booklet.

The activities include choices and pathways. If a student is having difficulty, you may need to encourage that student to work on all the choices rather than one. This would provide additional instruction and practice in a variety of ways.

Another form of support is routine contact with each individual. This might be achieved with a biweekly conference scheduled by you, or as students reach a certain point (e.g., after each section is completed), they may be directed to come to the conference area.

Special counselling may be needed to help students through difficult stages. Praise and encouragement are important motivators, particularly for those students who are not used to working independently.

Direct teaching may be needed and scheduled at certain points in the program. This might involve small groups or a large group. It might be used to take advantage of something timely (e.g., election, eclipse, etc.) or something prescheduled like the demonstration of a process, or involving students in a hands-on, practical experience.

Support at a distance might include tutoring by phone, teleconferencing, faxing, or planned visits. These contacts are the lifeline between learners and distance education teachers, so a warm dialogue is essential.

Management

Good management of an open-learning system is essential to the success of the program. The following areas need action to ensure that the system runs smoothly:

- Scheduling, Distributing, and Managing Resources – As discussed earlier, this may require a need for centres or a system for students to project and reserve the necessary resources.

- Scheduling Students – Students and teachers should work together to establish goals, course completion timelines, and daily timelines. Although students may push to continue for long periods of time (e.g., all morning), teachers should discourage this. Concentration, retention, and motivation is improved by taking scheduled breaks.

- Monitoring Student Progress – You will need to record when units are completed by each student. Your data might also include the projected date of completion if you are using a student contract approach.

Sample of a Student Progress Chart

Mathematics 30		Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Final Test
Billy Adams	P								
	A								
Louise Despins	P								
	A								
Violet Klassian	P								
	A								
P = Projected Completion Date				A = Actual Completion Date					

The student could keep a personal log as well. Such tracking of data could be stored easily on a computer.

- Recording Student Assessments – You will need to record the marks awarded to each student for work completed in each unit assignment booklet. The marks from these assignment booklets will contribute to a portion of the student's final mark. Other criteria may also be added (a special project, effort, attitude, etc.). Whatever the criteria, they should be made clear to all students at the beginning.

Sample of a Student's Assessment Chart

Mathematics 30	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Year's Average	Final Test	Final Mark
Billy Adams	67	65	54	47	78	67	56	62		
Louise Despins	43	50	54	55	48	42	44	48		
Violet Klassian	65	65	66	66	67	69	57	65		

Letter grading could easily be substituted.

- Recording Effectiveness of System – Keep ongoing records of how the system is working. This will help you in future planning.

Sample of a System Assessment Chart

Unit 1			
Date	Unit Booklet	Assignment Booklet	Resources/Media

The Role of the Teacher in an Open-Learning Classroom

The teachers in a conventional classroom spend a lot of time talking to large groups of learners. The situation in open learning requires a different emphasis. Teachers will probably meet learners individually or in very small groups.

With this approach it is necessary to move beyond the idea of a passive learner depending largely on a continually supportive teacher. The teacher must aim to build the student's confidence, to stimulate the learner into self-reliance, and to guide the learner to take advantage of routes that are most meaningful and applicable to the learner.

These materials are student-centred, not teacher-centred. The teacher needs to facilitate learning by providing general support to the learner.

Evaluation

Evaluation is important to the development of every learner. Data gathering and processing, and decision making, at the student and teacher level, serve as means of identifying strengths and weaknesses.

These specially designed learning packages contain many kinds of informal and formal evaluation.

Observation

In the classroom the teacher has the opportunity to see each student perform every day and to become aware of the level and nature of each student's performance.

Observations are more useful if they are recorded in an organized system. The following list of questions is a sample of types of observations and how they can be collected.

Observation Checklist

B. Adams / L. Despins / V. Klassian / H. Smith / K. Dailey

1. Does the student approach the work in a positive manner?

2. Is the student struggling with the reading level?

3. Does the student make good use of time?

4. Does the student apply an appropriate study method?

5. Can the student use references effectively, etc.?

Observation may suggest a need for an individual interview with a student.

Individual Conferences

Individual conferences may be paced (scheduled) by the calendar or at certain points in the unit, or they may be set up only as needed or requested.

During these conferences teachers can determine the student's progress and can assess the student's attitudes toward the subject, the program, school, and self, as well as the student's relationship with other students. With guided questions the teacher can encourage oral self-assessment; the student can discuss personal strengths or weaknesses in regard to the particular topic, unit, or subject area.

Self-Appraisal

Self-appraisal helps students recognize their own strengths and weaknesses. Through activities that require self-assessment, students also gain immediate feedback and clarification at early stages in the learning process. Teachers need to promote a responsible attitude toward these self-assessment activities. Becoming effective self-assessors is a crucial part of becoming an autonomous learner. By instructing, motivating, providing positive reinforcement, and systematically supervising, the learning facilitator will help students develop a positive attitude toward their own progress.

For variation, students may be paired and peer-assessing may become part of the system. The teacher may decide to have the student self-assess some of the activities, have a peer assess other activities, and become directly involved in assessing the remainder of the activities.

When the activities have been assessed, the student should be directed to make corrections. This should be made clear to students right from the start. It is important to note the correct association between the question and the response to clarify understanding, aid retention, and be of use for study purposes.

Many of the activities include choices for students. If the student is having difficulty, more practice may be warranted, and the student may need to be encouraged to do more of the choices.

Each topic within a unit includes additional types of activities called Extra Help and Extensions. Students are expected to be involved in the decision as to which pathway best suits their needs. They may decide to do both.

Self-appraisal techniques can also be introduced at the individual conferences. Such questions as the following might be included:

- What steps are you taking to improve your understanding of this topic?
- What method of study do you use most?
- How do you organize your material to remember it?
- What steps do you follow when doing an assignment in your assignment booklet?
- What could you do to become an even better reader?
- Do you have trouble following directions?
- Did you enjoy this unit?

A chart or checklist could be used for recording responses.

Informal Evaluation: Assignments

Informal evaluation, such as the assignments included in each unit, are an invaluable aid to the teacher. They offer ongoing assessment information about the student's achievement and the behaviour and attitudes that affect that achievement.

Each unit contains a separate booklet called the Assignment Booklet. This booklet assesses the knowledge or skills that the student has gained from the unit. **The student's mark for the unit may be based solely on the outcome of learning evident in the assignment booklet; however, you may decide to establish a value for other variables such as attitude or effort.** It is important that you establish at the beginning which outcomes will be evaluated and that all students clearly understand what is expected.

Final Test

All LFMs include a formal final test which can be photocopied for each member of the class. The test, closely linked to the learning outcomes stated in the unit booklets, give the teacher precise information concerning what each student can or cannot do. Answers and explanations and marking guides are also included.

The value of the final test and each unit is the decision of the classroom teacher. Following is a suggestion only.

Unit 1 12%	Unit 2 10%	Unit 3 13%
Unit 4 15%	Unit 5 20%	Unit 6 18%
	Unit 7 12%	
	Final Test 50%	

Introducing Students to the System

Your initiation to these learning materials began with a basic survey of what was included and how the components varied. This same process should be used with the class. After the materials have been explored, a discussion might include the advantages and the disadvantages of learning independently or in small groups. The roles of the students and teacher should be analyzed. The necessary progress checks and rules need to be addressed. Your introduction should motivate students and build a responsible attitude toward learning autonomously.

Skill Level

It is important for students to understand that there are certain skills that they will need in order to deal successfully with the course materials. They are listed below:

- understanding and using instructional materials (table of contents, index, list of illustrations, and appendices)

- interpreting graphs and charts

- using reference materials

- recognizing special symbols

- using a scientific calculator

Other general skills are using reliable study methods, outlining, and learning to read at a flexible rate.

To decide the level and amount of instruction needed to accommodate the varied levels among students, you may wish to prepare and administer skill inventories or pretests. If most students need help with a particular skill, you may want to plan a total class instructional session. If only certain students lack a skill, you may want to set up a temporary skill group to help students who need it, or you may want to develop a skills file for this purpose.
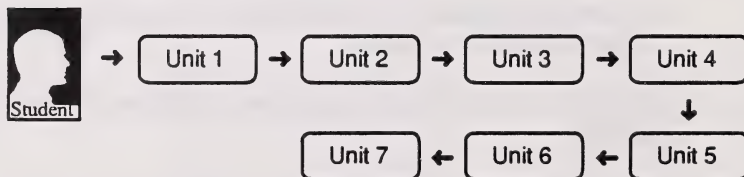
Reading Level

These course materials are largely print based, but poorer readers need not be discouraged. It is important that you assure the students that these materials have been designed for easy reading. The authors have employed special strategies that lower and control the reading level. Some of them are

- the conscious selection of vocabulary and careful structuring of sentences to keep the materials at an independent reading level

- the integration of activities, examples, and illustrations to break text into appropriate-sized chunks

- the inclusion of many kinds of organizers (advance, graphic, intermediate, concept mapping, post organizers) to help give students a structure for incorporating new concepts
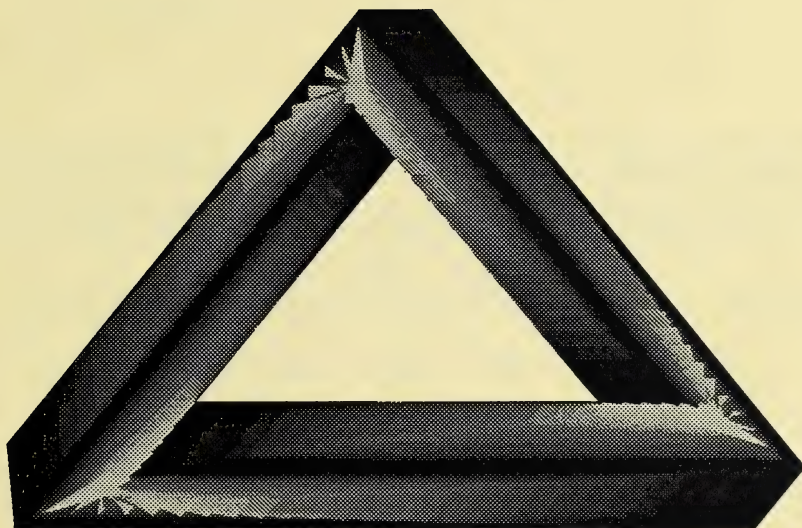
- the recognition that vocabulary and concepts are basic to understanding content materials and thus, must be handled systematically (defined in context, marginal notes, footnotes, and often in a specialized glossary)

- the acknowledgement that background knowledge and experience play a vital role in comprehension

- the systematic inclusion of illustrations and optional videos to help poorer readers and visual learners, and audiocassettes and software as an alternative to print-based learning

- a variety of formats (paragraphs, lists, charts, etc.) to help poorer readers who do not absorb or retain main ideas easily in paragraph format

- the inclusion of media pathways and activity choices to encourage an active rather than passive approach

- instruction in a meaningful setting rather than in a contrived, workbook style

- using purposeful reading, viewing, and doing to produce better interpretation of the course materials

- the recognition that students need structured experiences when reading, viewing, or listening to instructional materials: developing pupil readiness, determining the purpose, providing guided instruction and feedback, rereading if necessary, and extending (This structure closely resembles the reading process.)

To help make the learning package more readable, you can begin in your unit preparation by reading (viewing, listening to) all the related materials that are going to be used. You need a solid background in order to assess and develop a background knowledge for students. The students' experiential bases may be assessed through brainstorming sessions concerning the topic, having students write down all the ideas they have about the topic, or by using visuals and guided questions to predict what the topic might be about.

It is recommended that you start with Unit 1 because this unit includes basic introductory information, and it is also recommended that you end with Unit 7.

Student → Unit 1 → Unit 2 → Unit 3 → Unit 4 ↓ Unit 7 ← Unit 6 ← Unit 5

MATHEMATICS 30

LEARNING FACILITATOR'S MANUAL

UNIT 1

Distance Learning

Alberta EDUCATION

Note

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Topic 1: Introduction to Polynomial Functions

①   1.   Is the following polynomial a function?

$$P(x) = \sqrt{3}x^4 - 5x^2 + 3$$

Yes

②   2.   a.   Classify the polynomial $2x^2 - 3x^4 + \sqrt{7}$ according to degree.

       a.   The term with the highest degree in a polynomial determines the degree of the polynomial. The degree of the polynomial is **4** since the term $-3x^4$ has a degree of 4.

       b.   Classify the polynomial $2x^2 - 3x^4 + \sqrt{7}$ according to coefficients.

         This is a **real** polynomial since the coefficients 2, -3, and $\sqrt{7}$ belong to the real number system. The term $\sqrt{7}$ is irrational and it is a member of the real number system.

③   3.   If $P(x) = 4x^3 - 5x^2 + 3$, and $D(x) = x - 2$, then find $Q(x)$ and R by long division.

$$
\begin{array}{r}
4x^2 + 3x + 6 \\
x-2 \overline{\smash{)}4x^3 - 5x^2 + 0x + 3} \\
\underline{4x^3 - 8x^2} \\
3x^2 + 0x \\
\underline{3x^2 - 6x} \\
6x + 3 \\
\underline{6x - 12} \\
15
\end{array}
$$

$$Q(x) = 4x^2 + 3x + 6$$
$$R = 15$$

④ 4. a. Use synthetic division to determine the quotient and remainder when $6x^3 - 5x^2 - 9$ is divided by $2x - 3$.

Subtraction Method	Addition Method
Since $2x - 3 = 2\left(x - \frac{3}{2}\right)$, use $-\frac{3}{2}$ for the divisor.	Since $2x - 3 = 2\left(x - \frac{3}{2}\right)$, use $\frac{3}{2}$ for the divisor.

$$-\frac{3}{2}\,\bigg|\;\begin{array}{cccc} 6 & -5 & 0 & -9 \\ & -9 & -6 & -9 \\ \hline 6 & 4 & 6 & \boxed{0} \end{array}$$

$$\frac{3}{2}\,\bigg|\;\begin{array}{cccc} 6 & -5 & 0 & -9 \\ & 9 & 6 & +9 \\ \hline 6 & 4 & 6 & \boxed{0} \end{array}$$

To obtain the quotient, divide these coefficients by 2. Thus, $Q(x) = 3x^2 + 2x + 3$, and $R = 0$.

To obtain the quotient, divide these coefficients by 2. Thus, $Q(x) = 3x^2 + 2x + 3$, and $R = 0$.

b. If possible, express the polynomial $6x^3 - 5x^2 - 9$ as the product of two integral polynomials.

Since $R = 0, \left(x - \frac{3}{2}\right)$ is a factor of $6x^3 - 5x^2 - 9$.

$$P(x) = \left(x - \tfrac{3}{2}\right)\left(6x^2 + 4x + 6\right)$$

$$= \left(x - \tfrac{3}{2}\right)(2)\left(3x^2 + 2x + 3\right)$$

A factor of 2 is removed from each term in $6x^2 + 4x + 6$.

$$P(x) = (2x - 3)\left(3x^2 + 2x + 3\right)$$

The factor $\left(x - \frac{3}{2}\right)$ is multiplied by 2 to give the factor $(2x - 3)$.

Topic 1

_____ marks

Topic 2: The Remainder Theorem and Factor Theorem

③ 1. If $P(x) = 3x^3 - 8x^2 + 2x - 5$, calculate the exact value of the following:

  a. $P(-1)$

  b. $P(\sqrt{2})$

$$P(-1) = 3(-1)^3 - 8(-1)^2 + 2(-1) - 5$$
$$= -3 - 8 - 2 - 5$$
$$= -18$$

$$P(\sqrt{2}) = 3(\sqrt{2})^3 - 8(\sqrt{2})^2 + 2(\sqrt{2}) - 5$$
$$= 3(2\sqrt{2}) - 8(2) + 2\sqrt{2} - 5$$
$$= 6\sqrt{2} - 16 + 2\sqrt{2} - 5$$
$$= 8\sqrt{2} - 21$$

② 2. Using the remainder theorem, determine the remainder for the following division problem without actually performing the division.

$$(-2x^3 + 2x^2 + 7x - 5) \div (x - 3)$$

If a polynomial $P(x)$ is divided by $x - a$, the remainder, R, is equal to $P(a)$.

$$P(3) = -2(3)^3 + 2(3)^2 + 7(3) - 5$$
$$= -54 + 18 + 21 - 5$$
$$= -20$$

③ 3. Given the polynomial $P(x) = x^4 - dx^3 + 5x + 6$, when $P(x)$ is divided by $x + 2$, the remainder is 52. Determine the value of d.

$$P(-2) = 52$$
$$P(-2) = (-2)^4 - d(-2)^3 + 5(-2) + 6$$
$$= 16 + 8d - 10 + 6$$
$$P(-2) = 12 + 8d$$
$$52 = 12 + 8d$$
$$40 = 8d$$
$$d = 5$$

④

4. Use the factor theorem to determine if $x + 3$ is a factor of $2x^3 + 5x^2 - 4x - 3$. If $x + 3$ is a factor, then let the other factor be $ax^2 + bx + c$. State the values of a, b, and c.

A polynomial, $P(x)$, has the binomial $x - a$ as a factor if, and only if, $P(a) = 0$.

$$P(-3) = 2(-3)^3 + 5(-3)^2 - 4(-3) - 3$$
$$= -54 + 45 + 12 - 3$$
$$= 0$$

Therefore, $x + 3$ is a factor.

$$\begin{array}{r} 2x^2 - 1x - 1 \\ x+3\overline{)2x^3 + 5x^2 - 4x - 3} \\ \underline{2x^3 + 6x^2} \\ -1x^2 - 4x \\ \underline{-1x^2 - 3x} \\ -1x - 3 \\ \underline{-1x - 3} \\ 0 \end{array}$$

Therefore, $a = 2$, $b = -1$, and $c = -1$.

③

5. Use the factor theorem to determine if $x - 2$ is a factor of the polynomial $2x^4 - 5x^3 + 3x^2 - 4x + 4$. If the binomial is a factor of $P(x)$, then give a zero of the polynomial.

The binomial $x - 2$ is a factor if $P(2) = 0$.

$$P(2) = 2(2)^4 - 5(2)^3 + 3(2)^2 - 4(2) + 4$$
$$= 32 - 40 + 12 - 8 + 4$$
$$= 0$$

Thus, $x - 2$ is a factor.
The number a is a zero of a polynomial $P(x)$ if, and only if, $P(a) = 0$.
Since $P(2) = 0$, then 2 is a zero of the polynomial.

⑤   6.   Given $P(x) = 3x^3 - dx + c$, if $P(x)$ is divided by $x - 1$, it gives a remainder of -187. If $P(x)$ is divided by $x + 4$, it gives a remainder of -2. Determine the values of d and c.

$$P(x) = 3x^3 - dx + c$$

If a polynomial $P(x)$ is divided by $x - a$, the remainder, R, is equal to $P(a)$.

If the divisor is $(x - 1)$, then $P(1) = -187$.

$$P(1) = 3(1)^3 - d(1) + c$$
$$P(1) = 3 - d + c$$

Since $P(1) = -187$,

then $3 - d + c = -187$.

$$-d + c = -190 \quad ①$$

If the divisor is $(x + 4)$, then $P(-4) = -2$.

$$P(-4) = 3(-4)^3 - d(-4) + c$$
$$P(-4) = -192 + 4d + c$$

Since $P(-4) = -2$,

then $-192 + 4d + c = -2$.

$$4d + c = 190 \quad ②$$

$$① - ② \quad -5d = -380$$
$$d = 76$$

From ① $-76 + c = -190$

$$c = -114$$

The value of d is 76 and the value of c is -114.

7. a. For the polynomial $P(x) = 3x^3 - 2x^2 - 19x - 6$, show that $x + 2$ is a factor.

If $x + 2$ is a factor, then $P(-2) = 0$.

$$P(-2) = 3(-2)^3 - 2(-2)^2 - 19(-2) - 6$$
$$P(-2) = -24 - 8 + 38 - 6$$
$$P(-2) = 0$$

Thus, $x + 2$ is a factor.

b. Express the polynomial $P(x)$ as the product of three binomials each of the first degree.

$$
\require{enclose}
\begin{array}{r}
3x^2 - 8x - 3 \\
x+2 \enclose{longdiv}{3x^3 - 2x^2 - 19x - 6} \\
\underline{3x^3 + 6x^2} \\
-8x^2 - 19x \\
\underline{-8x^2 - 16x} \\
-3x - 6 \\
\underline{-3x - 6} \\
0
\end{array}
$$

$$P(x) = (x + 2)(3x^2 - 8x - 3)$$
$$P(x) = (x + 2)(3x + 1)(x - 3)$$

c. State the three zeros of the polynomial.

The zeros are -2, $-\frac{1}{3}$, and 3.

Topic 2

_____ marks

Topic 3: Factoring and Zeros

①   1.   List all the potential integral zeros of $P(x) = 2x^5 - 3x^4 - x^3 + 4x^2 + 5x - 16$.

The potential integral zeros are factors of the constant term -16.

The potential integral zeros are $\pm1, \pm2, \pm4, \pm8,$ and ±16.

②   2.   Determine all the integral zeros of $P(x) = x^3 + 3x^2 - x - 3$.

The potential integral zeros are ±1 and ±3, since these are factors of the constant term -3.
If $P(a) = 0$, then a is a zero.

$$P(1) = 1^3 + 3(1)^2 - 1 - 3$$
$$= 1 + 3 - 1 - 3$$
$$= 0. \text{ Thus, } 1 \text{ is a zero.}$$

$P(-1) = -1 + 3 + 1 - 3 = 0.$ Therefore, -1 is a zero.

$P(3) = 27 + 27 - 3 - 3 \neq 0.$ Thus, 3 is not a zero.

$P(-3) = -27 + 27 + 3 - 3 = 0.$ Therefore, -3 is a zero.

The integral zeros are $1, -1,$ and -3.

②   3.   For the polynomial $P(x) = 3x^3 - 2x^2 + 6x - 8$, determine the potential rational zeros.

The potential numerators are factors of the constant term -8.

The potential denominators are factors of the coefficient of the highest degree term. This coefficient is 3.

The potential rational zeros are all possible combinations of these factors when the potential numerators are divided by the potential denominators.

The potential numerators are $\pm1, \pm2, \pm4,$ and ±8.

The potential denominators are ±1 and ±3.

The potential rational zeros are $\pm1,\ \pm2,\ \pm4,\ \pm8,\ \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3},$ and $\pm\frac{8}{3}$.

③ 4. Determine all the rational zeros of $P(x) = 3x^3 + x^2 - 8x + 4$.

The potential numerators are factors of the constant term 4.

The potential numerators are ± 1, ± 2, and ± 4.

The potential denominators are factors of the coefficient of the highest degree term. This coefficient is 3.

The potential denominators are ± 1, and ± 3.

The potential rational zeros are all possible combinations of these factors when the potential numerators are divided by the potential denominators.

The potential rational zeros are ± 1, ± 2, ± 4, $\pm \frac{1}{3}$, $\pm \frac{2}{3}$, and $\pm \frac{4}{3}$.

$P(1) = 3 + 1 - 8 + 4 = 0$. Therefore, 1 is a zero.

Use -1 as the divisor in the synthetic division.

$$
\begin{array}{r|rrrr}
-1 & 3 & 1 & -8 & 4 \\
   &   & -3 & -4 & 4 \\
\hline
   & 3 & 4 & -4 & \boxed{0}
\end{array}
$$

Since the remainder is zero, $x - 1$ is a factor, and 1 is a zero. This checks with the direct substitution.

$P(x) = (x - 1)(3x^2 + 4x - 4)$

Now factor $3x^2 + 4x - 4 = (3x - 2)(x + 2)$.

$$\text{Let } (3x - 2) = 0 \quad \text{or} \quad x + 2 = 0.$$
$$3x = 2 \qquad\qquad x = -2$$
$$x = \tfrac{2}{3}$$

The rational zeros are 1, $\frac{2}{3}$, and -2.

5. Factor the integral polynomial $P(x) = x^4 - 7x^3 + 15x^2 - 11x + 2$ over the integers and determine all of its real zeros.

$P(1) = 1 - 7 + 15 - 11 + 2 = 0$. Therefore, 1 is a zero.

Use -1 as the divisor in the synthetic division procedure (subtraction method).

$$\begin{array}{r|rrrrr} -1 & 1 & -7 & 15 & -11 & 2 \\ & & -1 & 6 & -9 & 2 \\ \hline & 1 & -6 & 9 & -2 & \boxed{0} \end{array}$$

Since the remainder is zero, $x - 1$ is a factor and 1 is a zero.
$P(x) = (x-1)(x^3 - 6x^2 + 9x - 2)$

Use the synthetic division procedure on the coefficients $1, -6, 9,$ and -2.

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 9 & -2 \\ & & 1 & -7 & 16 \\ \hline & 1 & -7 & 16 & \boxed{-18} \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 9 & -2 \\ & & 2 & -16 & 50 \\ \hline & 1 & -8 & 25 & \boxed{-52} \end{array}$$

Since the remainder is not zero, -1 is not a zero.

Since the remainder is not zero, -2 is not a zero.

$$\begin{array}{r|rrrr} -2 & 1 & -6 & 9 & -2 \\ & & -2 & 8 & -2 \\ \hline & 1 & -4 & 1 & \boxed{0} \end{array}$$

Since the remainder is zero, $x - 2$ is a factor and 2 is a zero.

$P(x) = (x-1)(x-2)(x^2 - 4x + 1)$

Since $x^2 - 4x + 1$ cannot be factored any further, the factored form of $P(x)$ over the integers is $P(x) = (x-1)(x-2)(x^2 - 4x + 1)$. The zeros of $x^2 - 4x + 1$ can be determined by using the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{12}}{2}$$

$$= \frac{4 \pm 2\sqrt{3}}{2}$$

$$= 2 \pm 1\sqrt{3}$$

The real zeros are $1, 2, 2 + \sqrt{3},$ and $2 - \sqrt{3}$.

⑤   6.   Solve $12x^3 + 17x^2 + 2x - 3 = 0$.

Solving this equation for its roots is similar to determining the zeros of
$P(x) = 12x^3 + 17x^2 + 2x - 3$.

$P(1) = 12 + 17 + 2 - 3 \neq 0$. Thus, 1 is not a root.
$P(-1) = -12 + 17 - 2 - 3 = 0$. Thus, -1 is a root.
Use 1 as the divisor in the synthetic division procedure (subtraction method).

$$
\begin{array}{r|rrrr}
1 & 12 & 17 & 2 & -3 \\
  &   & 12 & 5 & -3 \\
\hline
  & 12 & 5 & -3 & \boxed{0}
\end{array}
$$

Since the remainder is 0, $x + 1$ is a factor, and -1 is a root.

The factored form is $(x+1)(12x^2 + 5x - 3) = 0$.

Now factor $12x^2 + 5x - 3$.

$$12x^2 + 5x - 3 = (4x + 3)(3x - 1)$$

Let $4x + 3 = 0$.   Let $3x - 1 = 0$.

$\qquad\qquad 4x = -3 \qquad\qquad 3x = 1$

$\qquad\qquad x = -\tfrac{3}{4} \qquad\qquad x = \tfrac{1}{3}$

The roots are -1, $-\tfrac{3}{4}$, and $\tfrac{1}{3}$.

⑥   7.   Solve $x^4 - 7x^3 + 15x^2 - 7x - 6 = 0$.

Potential rational zeros are ± 1, ± 2, ± 3, and ± 6.

$P(1) = 1 - 7 + 15 - 7 - 6 \neq 0$. Thus, 1 is not a zero.

$P(-1) = 1 + 7 + 15 + 7 - 6 \neq 0$. Thus, -1 is not a zero.

$P(2) = 16 - 56 + 60 - 14 - 6 = 0$. Therefore, 2 is a zero.

Use -2 as the divisor in the synthetic division (subtraction method).

$$
\begin{array}{r|rrrrr}
-2 & 1 & -7 & 15 & -7 & -6 \\
   &   & -2 & 10 & -10 & -6 \\
\hline
   & 1 & -5 & 5 & 3 & \boxed{0}
\end{array}
$$

Since the remainder is 0, $x - 2$ is a factor, and 2 is a zero.

$$P(x) = (x - 2)\left(x^3 - 5x^2 + 5x + 3\right)$$

Now apply the synthetic division procedure to the coefficients $1, -5, 5$, and 3 from the previous step.

$$
\begin{array}{r|rrrr}
2 & 1 & -5 & 5 & 3 \\
  &   & 2 & -14 & 38 \\
\hline
  & 1 & -7 & 19 & \boxed{-35}
\end{array}
$$

Since the remainder is not zero, -2 is not a zero.

$$
\begin{array}{r|rrrr}
3 & 1 & -5 & 5 & 3 \\
  &   & 3 & -24 & 87 \\
\hline
  & 1 & -8 & 29 & \boxed{-84}
\end{array}
$$

Since the remainder is not zero, -3 is not a zero.

(There is more room for your answer on the next page.)

7. (continued)

$$
\begin{array}{r|rrrr}
-3 & 1 & -5 & 5 & 3 \\
 & & -3 & 6 & 3 \\
\hline
 & 1 & -2 & -1 & \boxed{0}
\end{array}
$$

Since the remainder is 0, $x - 3$ is factor and 3 is a zero.

$$P(x) = (x-2)(x-3)\left(x^2 - 2x - 1\right)$$

Since $x^2 - 2x - 1$ cannot be factored, use the quadratic formula to determine the last two zeros.

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} \\
&= \frac{2 \pm \sqrt{4 + 4}}{2} \\
&= \frac{2 \pm \sqrt{8}}{2} \\
&= \frac{2 \pm 2\sqrt{2}}{2} \\
&= 1 \pm \sqrt{2}
\end{aligned}
$$

All the zeros are $2, 3, 1 + \sqrt{2}$, and $1 - \sqrt{2}$.

⑤   8.   Determine the integral polynomial of degree 3 that has the zeros 1, -3, and $\frac{1}{5}$, and $P(2) = 90$.

If 1 is a zero, $x - 1$ is a factor.

If -3 is a zero, $x - (-3) = x + 3$ is a factor.

If $\frac{1}{5}$ is a zero, $\left(x - \frac{1}{5}\right)$ is a factor. Thus, use $5x - 1$ for a factor.

$P(x) = k(x - 1)(x + 3)(5x - 1)$

$P(2) = 90$. Thus, substitute 2 for x on the right side of the equation and equate the left side of the equation to 90.

$90 = k(1)(5)(9)$

$90 = 45k$

$k = 2$

$P(x) = 2\left(5x^3 + 9x^2 - 17x + 3\right)$

$P(x) = 10x^3 + 18x^2 - 34x + 6$

⑤   9.   Determine the fourth-degree integral polynomial with zeros $\frac{3}{5}$, $-\frac{1}{2}$, and -1 (multiplicity of 2) which has a y-intercept of 6.

If $\frac{3}{5}$ is a zero, $(5x - 3)$ is the factor.

If $-\frac{1}{2}$ is a zero, $(2x + 1)$ is the factor.

If -1 is a zero, $(x + 1)(x + 1)$ are the factors since -1 has a multiplicity of 2.

$P(x) = k(5x - 3)(2x + 1)(x + 1)(x + 1)$

$P(0) = 6$

$6 = k(0 - 3)(0 + 1)(0 + 1)(0 + 1)$

$6 = k(-3)$

$k = -2$

$P(x) = -2\left(10x^4 + 19x^3 + 5x^2 - 7x - 3\right) = -20x^4 - 38x^3 - 10x^2 + 14x + 6$

Topic 3

_____ marks

Topic 4: Graphing

(8)   1.   Sketch the graph of $P(x) = -(x+2)^2(x-1)(x-3)$ after completing the following questions.

   a.   The y-intercept of the graph is ____-12____.

   b.   The zeros of $P(x)$ are _____$1, 3,$ and -2__.

   c.   The degree of $P(x)$ is ____4____.

   d.   Is the initial coefficient a_n positive or negative? _____negative____

   e.   Does the graph begin by rising or falling to the right? In which quadrant does this occur? _____rising in Quadrant 3_____

   f.   Does the graph end by rising or falling to the right? In which quadrant does this occur? _____falling in Quadrant 4_____

   g.   Sketch the graph of the polynomial.
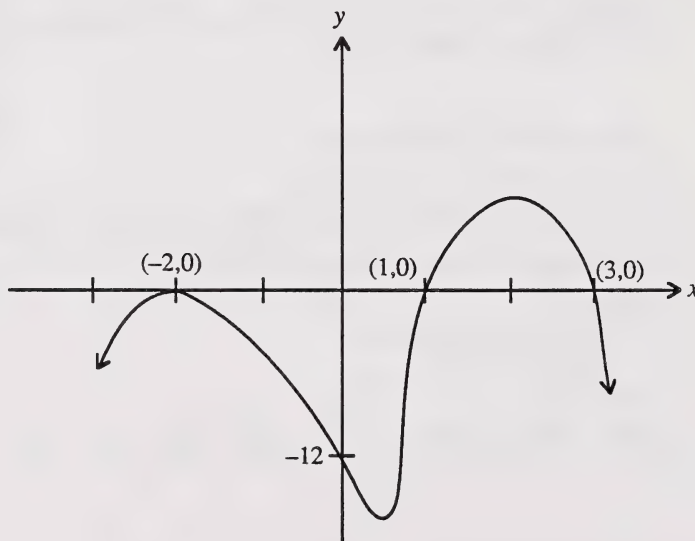
⑥ 2. Sketch the graph of $P(x) = -x^3 - x^2 + 8x + 12$.

The potential rational zeros are ±1, ±2, ±3, ±4, ±6, and ±12.

$P(1) = -1 - 1 + 8 + 12 \neq 0$. Thus, 1 is not a zero.
$P(-1) = 1 - 1 - 8 + 12 \neq 0$. Thus, −1 is not a zero.
$P(2) = -8 - 4 + 16 + 12 \neq 0$. Thus, 2 is not a zero.
$P(-2) = 8 - 4 - 16 + 12 = 0$. Therefore, −2 is a zero.

Use 2 as the divisor in the synthetic division method (subtraction).

$$
\begin{array}{r|rrrr}
2 & -1 & -1 & 8 & 12 \\
  &   & -2 & 2 & 12 \\
\hline
  & -1 & 1 & 6 & \boxed{0}
\end{array}
$$

Since the remainder is 0, $x + 2$ is a factor, and the zero is − 2.

$P(x) = (x+2)(-x^2 + 1x + 6)$.

Factor $(-x^2 + 1x + 6) = (-x - 2)(x - 3)$.

Let $(-x - 2) = 0$.   Let $x - 3 = 0$.
      $-2 = x$ $x = 3$

The zeros are 3 and − 2 (multiplicity of 2). The y-intercept is 12.

Since the initial coefficient is negative, the graph starts by falling to the right in Quadrant 2 and ends by falling to the right in Quadrant 4.
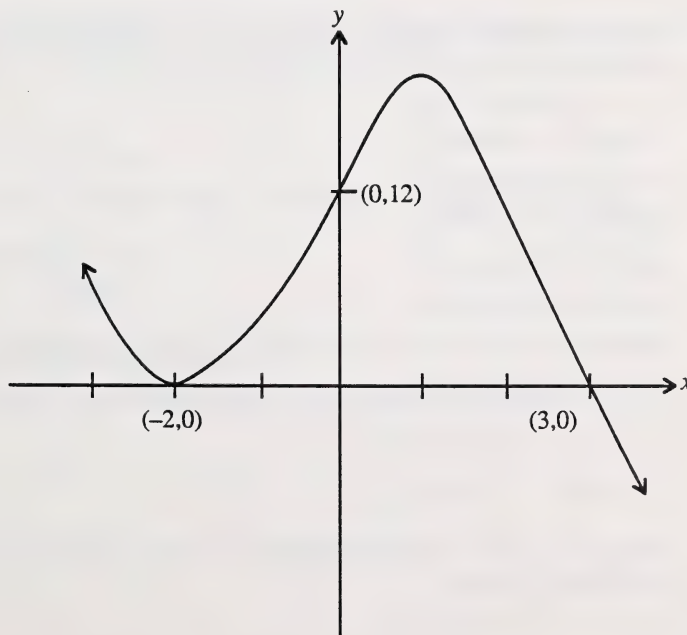
⑤

3. The graph of a polynomial function is shown below. State the polynomial function in factored form; then state it as an integral polynomial.
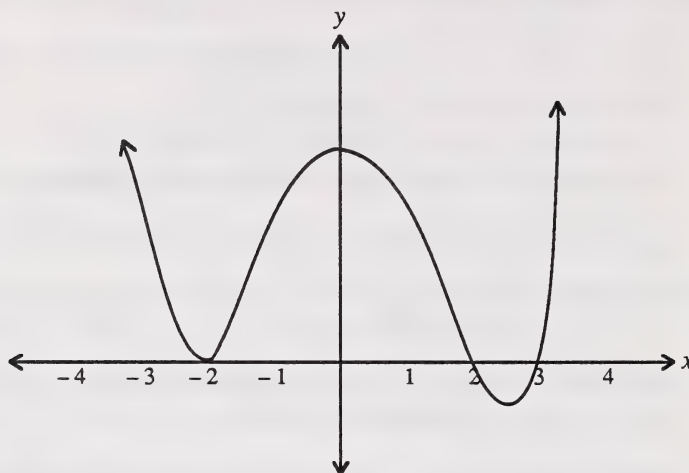
Since the graph touches at – 2, the zero – 2 has a multiplicity of 2. The graph intersects the x-axis at 2 and 3.

$$P(x) = (x+2)^2(x-2)(x-3)$$

$$P(x) = \left(x^2+4x+4\right)\left(x^2-5x+6\right)$$

$$P(x) = x^4 - x^3 - 10x^2 + 4x + 24$$

⑤

4. Determine the third-degree integral polynomial which crosses the x-axis at (– 1.5, 0) and touches the x-axis at (2, 0). The y-intercept is 12.

If – 1.5 is a zero, $\left(x+\frac{3}{2}\right)$ is a factor. A factor of $\left(x+\frac{3}{2}\right)$ is equivalent to a factor of $2x+3$ since both give a zero of – 1.5. Since the graph touches at (2, 0), the factors are $(x-2)^2$.

$$P(x) = (2x+3)(x-2)^2$$

$$= (2x+3)\left(x^2-4x+4\right)$$

$$P(x) = 2x^3 - 5x^2 - 4x + 12$$
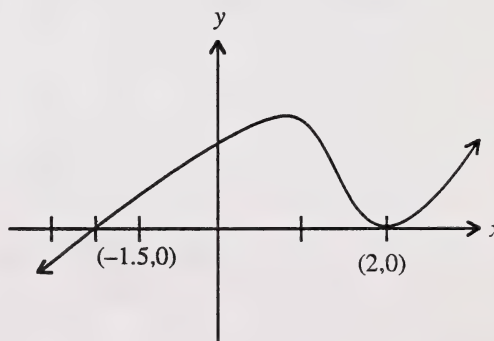
The graph must start rising in Quadrant 3 and ends rising in Quadrant 1. Thus, the initial coefficient is positive.

$$P(x) = 2x^3 - 5x^2 - 4x + 12$$

④ 5. Given the polynomial $P(x) = x^6 - 4x^5 - 2x^4 + x^3 + 3x^2 + 8x + 12$, determine the following:

a. the possible sums of multiplicities of the positive real zeros

$$x^6 - 4x^5 - 2x^4 + x^3 + 3x^2 + 8x + 12$$
+ to −   − to +

There are two changes in sign.
There can be 2 or 0 positive real
zeros.

Descartes' Rule
The number of positive real zeros
of a real polynomial is equal to the
number of changes in sign, or is
less than the number of changes by
an even integer.

b. the possible sums of multiplicities of the negative real zeros

Substitute $-x$ for x to obtain $P(-x)$. Then check the number of changes
in sign.

$$P(-x) = x^6 + 4x^5 - 2x^4 - x^3 + 3x^2 - 8x + 12$$
+ to −   − to +   + to − − to +

There are four changes in sign. There can
be 4, 2, or 0 negative real zeros.

Descartes' Rule
The number of negative real zeros
equals the number of changes in sign
of $P(-x)$, or is less than the number
of changes by an even integer.

② 6. Given the polynomial $P(x) = -2x^3 - 3x - 1$, what is the largest number of real zeros that the
polynomial may have?

$$P(x) = -2x^3 - 3x - 1$$

There are no changes in sign. Thus, there are no positive real zeros.

$$P(-x) = 2x^3 + 3x - 1$$
+ to −

There is one change in sign. Thus, there can be one negative real zero.

The largest number of real zeros that the polynomial may have is one.
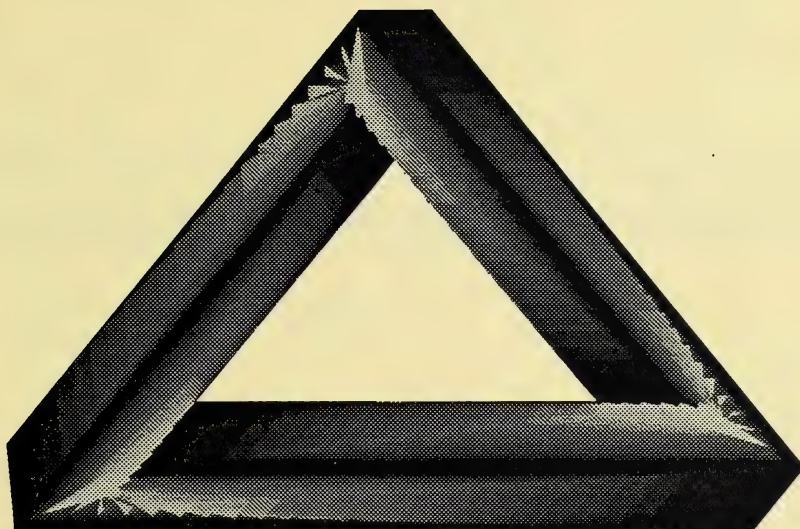
Topic 4

_____ marks

MATHEMATICS 30

LEARNING FACILITATOR'S MANUAL

UNIT 2

Distance Learning

Alberta
EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Topic 1: Exponential Functions

⟨2⟩ 1. Simplify the following expressions.

    a. $8^{\frac{2}{3}} \cdot 16^{-\frac{3}{4}}$

        $\left(2^3\right)^{\frac{2}{3}} \cdot \left(2^4\right)^{-\frac{3}{4}}$

        $2^{\frac{6}{3}} \cdot 2^{-\frac{12}{4}} = 2^2 \cdot 2^{-3} = 2^{-1}$ or $\frac{1}{2}$

    b. $\left(\dfrac{a^2 b^{-3}}{b^{-\frac{1}{2}}}\right)^4$

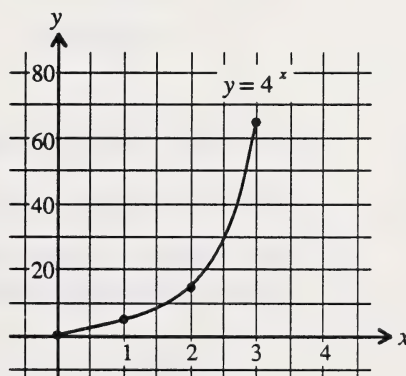 $= \dfrac{a^8 b^{-12}}{b^{-2}} = a^8 b^{-10}$

⟨2⟩ 2. Use the graph of $y = 4^x$ to approximate the unknown values. For your solutions, you must show the required lines on the graph.

    a.   For $y = 4^{2.7}$, approximate y to the nearest whole unit.

        $y \doteq 33$

    b.   For $10 = 4^x$, approximate x to one decimal place.

        $x \doteq 1.7$

⟨2⟩ 3. Solve for x.

$$9^{2x} = \left(3^x\right)^{x-1}$$
$$\left(3^2\right)^{2x} = \left(3^x\right)^{x-1}$$
$$3^{4x} = 3^{x^2 - x}$$

Equate the exponents since the bases are equal.

$$4x = x^2 - x$$
$$0 = x^2 - 5x$$
$$0 = x(x-5)$$
$$x = 0 \text{ or } x = 5$$

② 4. A certain kind of bacteria will grow to 32 times its initial amount in 40 minutes. Find the time required for the doubling period. (Use the formula $A = A_0 (2^{\frac{t}{p}})$, where A = multiple of the initial amount, A_0 = initial amount, t = time, and p = time required.)

$$A = A_0 (2^{\frac{t}{p}})$$
$$A_0 = 1, A = 32, t = 40$$
$$32 = 1(2^{\frac{40}{p}})$$
$$2^5 = 2^{\frac{40}{p}}$$

Equate the exponents since the bases are equal.

$$5 = \frac{40}{p}$$
$$p = 8 \text{ minutes}$$

② 5. In 8 days, a radioactive substance decays to $\frac{1}{\sqrt{2}}$ of its initial amount. Find the half-life of the substance. (Use the formula $N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$ where N = amount of substance remaining, N_0 = amount of substance when $t = 0$, t = time, and h is the half-life of the substance.)

The half-life is the time required for the substance to decay to one half of its original amount.

$$N = \frac{1}{\sqrt{2}} N_0 \text{ and } t = 8$$

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$\frac{1}{\sqrt{2}} N_0 = N_0 \left(\frac{1}{2}\right)^{\frac{8}{h}}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{2^{\frac{8}{h}}}$$

$$2^{-\frac{1}{2}} = 2^{-\frac{8}{h}}$$

$$-\frac{1}{2} = -\frac{8}{h}$$

$$h = 16$$

The half-life is 16 days.

Topic 1

_____ marks

Topic 2 : Introduction to Logarithms

② 1. For the exponential function $r = p^q$, answer the following:

  a. State the inverse.

  $q = p^r$

  b. Change the inverse from exponential to logarithmic form.

  $q = p^r \longleftrightarrow r = \log_p q$

③ 2. The graph below is the graph of $y = \log_3 x$.

  a. Use the graph to approximate
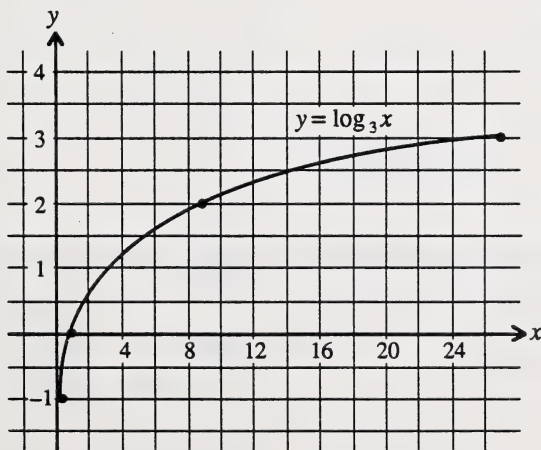 the value of $\log_3 6$ to 1 decimal place.

  $\log_3 6 \doteq 1.6$

  b. Approximate the value for $\log_3 20$
  to 1 decimal place.

  $\log_3 20 \doteq 2.7$

  c. Use the graph to approximate the
  value of x to the nearest whole unit
  when $y = 2.5$.

  $x \doteq 14$

③ 3. Express the following in logarithmic form.

   a. $64 = 4^3$

   $\log_4 64 = 3$

   b. $2^{-3} = \frac{1}{8}$

   $\log_2\left(\frac{1}{8}\right) = -3$

② 4. Express the following in exponential form.

   a. $\log_a b = c$

   $a^c = b$

   b. $\log_2\left(\frac{1}{32}\right) = -5$

   $\frac{1}{32} = 2^{-5}$

④ 5. Evaluate.

   a. $\log_3\left(\frac{1}{27}\right)$

   If $x = \log_3 \frac{1}{27}$,

   then $3^x = \frac{1}{27}$.

   $3^x = \frac{1}{3^3} = 3^{-3}$

   $\therefore x = -3$

   b. $\log_3 \sqrt[4]{81}^{\,3}$

   $\log_3 \sqrt[4]{81^3} = \log_3 3^3$

   $= 3$

② 6. Evaluate.

   a. $\log_5 5^7$

   $\log_5 5^7 = 7\log_5 5$

   $= 7(1)$

   $= 7$

   b. $6^{\log_6 14}$

   Since $a^{\log_a m} = m$,

   $6^{\log_6 14} = 14$.

(4) 7. Solve for the unknown.

a. $\log_x 64 = -3$

$x^{-3} = 64$

$\dfrac{1}{x^3} = \dfrac{64}{1}$

$x^3 = \dfrac{1}{64}$

$x = \dfrac{1}{4}$

b. $\log_{32} x = -\dfrac{3}{5}$

$(32)^{-\frac{3}{5}} = x$

$\dfrac{1}{(32)^{\frac{3}{5}}} = x$

$\dfrac{1}{2^3} = x$

$\dfrac{1}{8} = x$

(3) 8. Evaluate.

a. $\log_3(\log_4 64)$

$\log_4 64 = 3$

Thus, $\log_3 3 = 1.$

b. $\log_4\left(\log_3 3^2\right)$

Let $x = \log_4\left(\log_3 3^2\right).$

$x = \log_4 2$ since $\log_3 3^2$ equals 2.

$4^x = 2$

$\left(2^2\right)^x = 2^1$

$2x = 1$

$x = \dfrac{1}{2}$

Thus, $\log_4\left(\log_3 3^2\right) = \dfrac{1}{2}.$

(2) 9. Use your calculator to evaluate the following to five decimal places.

a. $\log 18$

1.255 27

b. $10^{2.46}$

288.403 15

Topic 2

_____marks

Topic 3 : Laws of Logarithms

④ 1. Use the laws of logarithms to expand the single logarithms.

    a. $\log_3\left(\dfrac{\sqrt[4]{AB}}{C}\right)$
             b. $\log_4\left(\dfrac{AB^{\frac{2}{3}}}{C^{\frac{1}{3}}}\right)$

        $=\dfrac{1}{4}\log_3 A+\dfrac{1}{4}\log_3 B-\log_3 C$
          $=\log_4 A+\dfrac{2}{3}\log_4 B-\dfrac{1}{3}\log_4 C$

⑤ 2. For each of the following, express as a single logarithm.

    a. $\dfrac{1}{4}\log_7 A+\dfrac{1}{2}\log_7 B-\dfrac{1}{6}\log_7 C=\log_7\left(\dfrac{A^{\frac{1}{4}}B^{\frac{1}{2}}}{C^{\frac{1}{6}}}\right)$

    b. $\dfrac{1}{3}\big[\log_5 A+2\log_5 B\big]-3\big[\log_5 C+\log_5 D\big]=\log_5\left[\dfrac{\left(AB^2\right)^{\frac{1}{3}}}{(CD)^3}\right]$ or $\log_5\left(\dfrac{\sqrt[3]{AB^2}}{C^3D^3}\right)$

⑥ 3. Express as a single logarithm in simplest form.

    a. $\log_4 8+2\log_4 3-\log_4 12$
             b. $2\log_3 7+3\log_3 2-2\log_3 2$

      $\log_4\left(\dfrac{8(3)^2}{12}\right)=\log_4\left(\dfrac{8\times9}{12}\right)=\log_4 6$
     $\log_3\left[\dfrac{7^2(2)^3}{2^2}\right]=\log_3\left(\dfrac{49\times8}{4}\right)=\log_3 98$

② 4. Evaluate $\log_3 5589-\log_3 69$.

    $\log_3\left(\dfrac{5589}{69}\right)=\log_3 81=4$

② 5. Express as the logarithm of a product or quotient; then use your calculator to evaluate to the nearest thousandth.

$\log 60.32 - \log 9.84 - \log 5.68$

$$\log \frac{(60.32)}{(9.84)(5.68)} = \log 1.079\ 239 = 0.033$$

⑤ 6. Evaluate.

a. $\log_2 10 + \log_2 3.2$

Let $x = \log_2 10 + \log_2 3.2$

$\quad = \log_2 (10 \times 3.2)$

$\quad = \log_2 (32)$

$\quad = 5.$

b. $\log_8 (2)(\sqrt{32})$

		Change of Base Method

$\log_8 2 + \log_8 32^{\frac{1}{2}}$

If $x = \log_8 2$, then $8^x = 2$.

$(2^3)^x = 2^1$

Equate the exponents since the bases are equal.

$3x = 1$

$x = \frac{1}{3}$

$\log_8 32^{\frac{1}{2}} = \frac{1}{2}\log_8 32$

If $y = \log_8 32$,

then $8^y = 32$.

$(2^3)^y = 2^5$

$3y = 5$

$y = \frac{5}{3}$

$\frac{1}{2}\log_8 32 = \frac{1}{2}\left(\frac{5}{3}\right)$

$\quad = \frac{5}{6}$

$\log_a b = \frac{\log_c b}{\log_c a}$

where c equals 10.

$\log_8 (2)(\sqrt{32}) = \frac{\log_{10}(2)(\sqrt{32})}{\log_{10} 8}$

$\quad = \frac{1.053\ 605}{0.903\ 090}$

$\quad = 1.1666$

Thus, $\frac{1}{3} + \frac{5}{6} = \frac{7}{6}$.

⑶ 7. Given that $\log_5 3 = 0.682\ 606$, find the value of $\log_5 (15)^{\frac{2}{3}}$ to five decimal places .

$$\log_5 (15)^{\frac{2}{3}} = \frac{2}{3}\log_5 15$$

$$= \frac{2}{3}\log_5 (3\times5)$$

$$= \frac{2}{3}\left(\log_5 3+\log_5 5\right)$$

$$= \frac{2}{3}\left(0.682\ 606+1\right)$$

$$= \frac{2}{3}\left(1.682\ 606\right)$$

$$= 1.121\ 74$$

⑶ 8. Use your calculator to evaluate the following to three decimal places.

a. $\log\sqrt[5]{23.8}$

$$\frac{1}{5}\log 23.8 = \frac{1}{5}\left(1.376\ 576\ 9\right)$$

$$= 0.2753$$

$$= 0.275$$

b. $\log\left(8.7^{\frac{3}{5}}\times4.2^{\frac{1}{4}}\right)$

$$\frac{3}{5}\log 8.7+\frac{1}{4}\log 4.2$$

$$= 0.5637+0.1558$$

$$= 0.7195$$

$$= 0.720$$

Topic 3

_____marks

Topic 4 : Applications

④ 1. Solve the following exponential equations to five decimal places.

a. $5^{\frac{1}{3}x} = 18$

$$\frac{1}{3}x\log 5 = \log 18$$

$$x = \frac{3\log 18}{\log 5}$$

$$x = \frac{3.765\ 818}{0.698\ 970}$$

$$x = 5.387\ 67$$

b. $8^{4x} = 5^{2x+2}$

$$4x\log 8 = (2x+2)\log 5$$

$$4x\log 8 = 2x\log 5 + 2\log 5$$

$$4x\log 8 - 2x\log 5 = 2\log 5$$

$$x(4\log 8 - 2\log 5) = 2\log 5$$

$$x = \frac{2\log 5}{4\log 8 - 2\log 5}$$

$$x = \frac{1.397\ 940}{3.612\ 360 - 1.397\ 940}$$

$$= 0.631\ 29$$

② 2. Use the change of base formula to find $\log_6 72$ correct to five decimal places.

$$\log_a b = \frac{\log_c b}{\log_c a} \quad \text{where } c \text{ equals } 10.$$

$$\log_6 72 = \frac{\log 72}{\log 6} = \frac{1.857\ 332}{0.778\ 151} = 2.386\ 85$$

③ 3. Solve $x^{\frac{2}{3}} = 24$ and state the answer to three decimal places.

$$\log x^{\frac{2}{3}} = \log 24$$

$$\frac{2}{3}\log x = \log 24$$

$$\log x = \frac{3}{2}\log 24$$

$$= \frac{3}{2}(1.380\ 211\ 242)$$

$$\log x = 2.070\ 316\ 863$$

$$x = 117.576$$

③ 4. Solve for x.

$$\log(x) - \log 4 = \log(7x) + \log(2x)$$

$$\log\left(\frac{x}{4}\right) = \log(7x)(2x)$$

$$\frac{x}{4} = 14x^2$$

$$56x^2 = x$$

$$56x^2 - x = 0$$

$$x(56x - 1) = 0$$

$$x = 0 \text{ or } x = \tfrac{1}{56}$$

The value $x = 0$ is inadmissible. Thus, $x = \tfrac{1}{56}$.

③ 5. Solve for x.

$$\log_4(x-1) + \log_4(x+2) = 1$$

$$\log_4(x-1)(x+2) = 1$$

$$\log_4(x-1)(x+2) = \log_4 4 \qquad \log_a a = 1$$

If $\log_x a = \log_x b$, then $a = b$.

Thus, $(x-1)(x+2) = 4$.

$$x^2 + x - 2 = 4$$

$$x^2 + x - 6 = 0$$

$$(x-2)(x+3) = 0$$

$$x - 2 = 0 \text{ or } x + 3 = 0$$

$$x = 2 \text{ or } x = -3$$

Thus, $x = 2$ since $x = -3$ is inadmissible.

② 6. Solve $e^x = 12.7$ to 6 decimal places.

$$\ln e^x = \ln 12.7$$

$$x \ln e = \ln 12.7$$

$$x(1) = \ln 12.7$$

$$x \doteq 2.541\,602$$

② 7. Solve $\ln x = -3$ to 6 decimal places.

$\log_e x = -3 \longleftrightarrow e^{-3} = x$

3 $\boxed{+/-}$ $\boxed{\text{INV}}$ $\boxed{\ln}$ gives 0.049 787.

④ 8. The population of a small city was 41 300 in 1976 and 47 700 in 1984. Predict the population for 1995. Assume exponential growth and state the final answer to the nearest thousand.

A formula for predicting population growth is $G = G_0\left(2^{cy}\right)$ where G is the population at time y in years, G_0 is the population when $y = 0$, and c is a constant that is always positive.

$$G = G_0\left(2^{cy}\right)$$

$$47\ 700 = 41\ 300\left(2^{c(8)}\right)$$

$$\frac{47\ 700}{41\ 300} = 2^{8c}$$

$$\log\left(\frac{47\ 700}{41\ 300}\right) = 8c\log 2$$

$$c = \frac{\log\left(\dfrac{47\ 700}{41\ 300}\right)}{8\log 2} = \frac{0.062\ 568}{2.408\ 240} = 0.0260$$

For the year 1995, $G_0 = 41\ 300$

$$c = 0.0260,\ y = 19$$

$$G = 41\ 300\left(2^{0.0260}\right)^{(19)}$$

$$G = 58\ 164$$

$$G = 58\ 000$$

④ 9. Determine the half-life of a radioactive substance if 200 g of this substance decays to 30 g in 40 days.

The half-life is the time required for the substance to decay to one half of its original amount.

The formula $a = a_0\left(2^{-\frac{t}{h}}\right)$ represents the amount (a) remaining after time (t).

The original amount is a_0 and h is the half-life of the substance.

$$a = a_0\left(2^{-\frac{t}{h}}\right)$$

When $a = 30, a_0 = 200, t = 40$, find h.

$$a = a_0\left(\frac{1}{2}\right)^{\frac{t}{h}} \qquad 2^{-\frac{t}{h}} = \frac{1}{2^{\frac{t}{h}}} = \left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$30 = 200\left(\frac{1}{2}\right)^{\frac{40}{h}}$$

$$\frac{30}{200} = \left(\frac{1}{2}\right)^{\frac{40}{h}}$$

$$\log\left(\frac{3}{20}\right) = \frac{40}{h}\log\left(\frac{1}{2}\right)$$

$$\log\left(\frac{3}{20}\right)(h) = 40\log\left(\frac{1}{2}\right)$$

$$h = \frac{40\log\left(\frac{1}{2}\right)}{\log\left(\frac{3}{20}\right)}$$

$$= \frac{-12.041\,199}{-0.823\,908}$$

$$= 14.6$$

The half-life is 14.6 days.

④   10. Find the amount that $1200 will produce if invested at 9% per year and compounded semiannually for 6 years.

P is the original amount deposited at a yearly rate of interest, (r), and compounded n times per year. A is the amount produced after t years.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 1200\left(1+\frac{0.09}{2}\right)^{2(6)}$$

$$A = 1200(1+0.045)^{12}$$

$$A = 1200(1.045)^{12}$$

$$\log A = \log 1200 + 12\log 1.045$$

$$= 3.079\ 181\ 246 + 12(0.019\ 116\ 29)$$

$$\log A = 3.308\ 576\ 731$$

$$A = \$2035.06 \quad \text{since } 10^{3.308\ 576\ 731} \text{ equals } 2035.057\ 717.$$

If $A = 1200(1.045)^{12}$ is calculated directly without using

the logarithm laws, the answer obtained is also $2035.06.

④   11. Earthquake A has a magnitude of 5.8 on the Richter Scale and earthquake B has a reading of 5.1. How many times greater is earthquake A than earthquake B?

The Richter Scale uses the formula $\log_{10}\dfrac{I_m}{I_r}$, where I_m is the intensity of the earthquake

being measured and I_r is the intensity of a reference earthquake.

Earthquake A   Earthquake B

$$5.8 = \log_{10}\frac{I_m}{I_r} \qquad\qquad 5.1 = \log_{10}\frac{I_m}{I_r}$$

$$10^{5.8} = \frac{I_m}{I_r} \qquad\qquad\qquad 10^{5.1} = \frac{I_m}{I_r}$$

$$I_m = 10^{5.8}\,I_r \qquad\qquad\qquad I_m = 10^{5.1}\,I_r$$

$$\frac{I_m \text{ for earthquake A}}{I_m \text{ for earthquake B}} = \frac{10^{5.8}\,I_r}{10^{5.1}\,I_r} = 10^{5.8-5.1} = 10^{0.7} = 5.01$$

Earthquake A is 5 times more intense than earthquake B.

Topic 4

_____ marks

MATHEMATICS 30

LEARNING FACILITATOR'S MANUAL

UNIT 3

Distance Learning

Alberta EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Mathematics 30
Learning Facilitator's Manual
Unit 3
Sequences, Series, Limits
Alberta Distance Learning Centre
ISBN No. 0-7741-0117-2

Topic 1: Sequences

② 1. Determine the first four terms of the sequence for the function $t_n = 2n^2 - 3n$.

$$t_1 = 2(1)^2 - 3(1) \qquad t_2 = 2(2)^2 - 3(2)$$
$$t_1 = 2 - 3 \qquad\qquad = 2(4) - 6$$
$$t_1 = -1 \qquad\qquad t_2 = 2$$

$$t_3 = 2(3)^2 - 3(3) \qquad t_4 = 2(4)^2 - 3(4)$$
$$= 18 - 9 \qquad\qquad = 32 - 12$$
$$t_3 = 9 \qquad\qquad t_4 = 20$$

The first four terms are -1, 2, 9, and 20.

③ 2. Determine a recursion formula for the sequence 3, 7, 11, ...

$$t_1 = 3$$
$$t_2 = 7 \quad \text{or} \quad t_2 = t_1 + 4$$
$$t_3 = 11 \quad \text{or} \quad t_3 = t_2 + 4$$

The nth term is 4 more than the $(n-1)$ term.

The recursion formula is

$$t_n = t_{n-1} + 4, \ \ n > 1, \ t_1 = 3 \quad \text{or} \quad t_{n+1} = t_n + 4, \ t_1 = 3 \ .$$

Topic 1

_____ marks

Topic 2 : Arithmetic Sequences

(2)

1. For the arithmetic sequence $20, 17, 14, 11, \ldots, -13$, determine the position of the last term.

$$t_n = a + (n-1)d$$
$$-13 = 20 + (n-1)(-3)$$
$$-13 = -3n + 23$$
$$3n = 36$$
$$n = 12$$

(4)

2. Determine t_{14} and the general term (t_n) of the arithmetic sequence $-16, -12, -8, \ldots$.

$$t_n = a + (n-1)d$$
$$t_{14} = -16 + (14-1)(4)$$
$$= -16 + (13)(4)$$
$$= -16 + 52$$
$$t_{14} = 36$$
$$t_n = a + (n-1)d$$
$$= -16 + (n-1)4$$
$$= -16 + 4n - 4$$
$$t_n = 4n - 20$$

(3)

3. Insert three arithmetic means between -2 and 30.

$$t_n = a + (n-1)d \qquad\qquad t_3 = t_2 + 8$$
$$30 = -2 + (5-1)d \qquad\qquad = 6 + 8$$
$$32 = 4d \qquad\qquad\qquad = 14$$
$$d = 8$$

$$t_2 = t_1 + 8 \qquad\qquad\quad t_4 = t_3 + 8$$
$$= -2 + 8 \qquad\qquad\quad = 14 + 8$$
$$= 6 \qquad\qquad\qquad\quad = 22$$

The three arithmetic means are 6, 14, and 22.

(5)   4.   An arithmetic sequence has $t_6 = -12$ and $t_{20} = -96$. Determine the common difference, the first term, and the general term.

$$t_n = a + (n-1)d \qquad\qquad t_n = a + (n-1)d$$
$$-12 = a + (6-1)d \qquad\qquad -96 = a + (20-1)d$$
$$-12 = a + 5d \;\; \textcircled{1} \qquad\qquad -96 = a + 19d \;\; \textcircled{2}$$

$\textcircled{2} - \textcircled{1} \qquad -84 = 14d$

$$d = -6$$

Substitute $d = -6$ in $\textcircled{1}$.
$$-12 = a + 5(-6)$$
$$-12 = a - 30$$
$$a = 18$$

$$t_n = a + (n-1)d$$
$$t_n = 18 + (n-1)(-6)$$
$$t_n = 18 - 6n + 6$$
$$t_n = -6n + 24$$

(3)   5.   If $1000 is loaned at 12% per annum simple interest, determine the money owing (loan plus interest) at the end of five years.

The money owing at the end of the first year is $1000 + $1000 (0.12).

$1000 + $120 = $1120

Thus, $a = \$1120$, $d = \$120$, and $n = 5$.
$$t_n = a + (n-1)d$$
$$= \$1120 + (5-1)\$120$$
$$= \$1120 + \$480$$
$$= \$1600$$

The money owing after five years is $1600.

③ 6. How many multiples of 6 are there between 75 and 290?

The multiple of 6 greater than 75 is 78 since 6 (13) = 78.
The multiple of 6 less than 290 is 288 since 6 (48) = 288.
The arithmetic sequence formed by these multiples of 6 is 78, 84, 90, . . . , 288.

$a = 78, \ t_n = 288, \ d = 6$

$$t_n = a + (n-1)d$$
$$288 = 78 + (n-1)(6)$$
$$288 = 78 + 6n - 6$$
$$288 = 72 + 6n$$
$$216 = 6n$$
$$n = 36$$

There are 36 multiples of 6 between 75 and 290.

Topic 2

_____marks

Topic 3: Geometric Sequences

③ 1. For the geometric sequence $4,\ 2\sqrt{2},\ 2,\ \ldots$, determine the common ratio. Then calculate the next two terms in the sequence.

$$r = \frac{t_3}{t_2}$$

$$r = \frac{2}{2\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

$$t_4 = t_3 \times r$$

$$t_4 = 2 \times \frac{\sqrt{2}}{2}$$

$$= \sqrt{2}$$

$$t_5 = t_4 \times r$$

$$= \sqrt{2} \times \frac{\sqrt{2}}{2}$$

$$= \frac{2}{2}$$

$$= 1$$

Thus, $t_4 = \sqrt{2}$ and $t_5 = 1$.

③　2.　Determine the general term and the ninth term of the geometric sequence $-6,\ 4, \frac{-8}{3}, \ldots \ .$

$$a = -6,$$

$$r = \frac{t_2}{t_1}$$

$$= \frac{4}{-6}$$

$$= \frac{-2}{3}$$

$$t_n = ar^{n-1}$$

$$t_n = (-6)\left(-\frac{2}{3}\right)^{n-1}$$

$$t_9 = (-6)\left(-\frac{2}{3}\right)^{9-1}$$

$$= (-6)\left(-\frac{2}{3}\right)^{8}$$

$$= \frac{-1536}{6561} \quad \text{or} \quad \frac{-512}{2187}$$

④ 3. Given the geometric sequence $-60, 120, -240, \ldots, 30\,720$, determine which term in the sequence is -3840 and calculate the number of terms in the sequence.

$$a = -60,$$

$$r = \frac{t_2}{t_1}$$

$$= \frac{120}{-60}$$

$$= -2$$

$$t_n = ar^{n-1}$$

$$-3840 = (-60)(-2)^{n-1}$$

$$64 = (-2)^{n-1}$$

$$(-2)^6 = (-2)^{n-1}$$

$$6 = n-1$$

$$n = 7$$

Thus, $t_7 = -3840$.

$$t_n = ar^{n-1}$$

$$30\,720 = (-60)(-2)^{n-1}$$

$$-512 = (-2)^{n-1}$$

$$(-2)^9 = (-2)^{n-1}$$

$$9 = n-1$$

$$n = 10$$

Thus, there are 10 terms in the sequence.

④ 4. In a geometric sequence, $t_7 = 1792$ and $t_{10} = -14\ 336$. Determine a, r, and t_n for the geometric sequence.

$$t_n = ar^{n-1}$$
$$t_7 = ar^{7-1}$$
$$1792 = ar^6$$
$$\frac{ar^9}{ar^6} = \frac{-14\ 336}{1792}$$
$$r^3 = -8$$
$$r = -2$$

$$t_n = ar^{n-1}$$
$$t_{10} = ar^{10-1}$$
$$-14\ 336 = ar^9$$

$$1792 = ar^6$$
$$1792 = a(-2)^6$$
$$1792 = a(64)$$
$$28 = a$$
$$t_n = ar^{n-1}$$
$$t_n = (28)(-2)^{n-1}$$

Thus, $r = -2$, $a = 28$, and $t_n = (28)(-2)^{n-1}$.

(6)   5.   A person deposits $1800 in a savings account which earns 8% per annum compounded quarterly.

    a.   What is the amount of this investment at the end of 5 years?

$$A = P(1+i)^n$$
$$A = 1800(1.02)^{20}$$
$$A = 1800(1.485\ 947\ 396)$$
$$A = \$2674.71$$

    b.   How much interest was earned on the deposit at the end of 5 years?

$$I = A - P$$
$$I = 2674.71 - 1800$$
$$I = \$874.71$$

    c.   If this person needs $2400 to make a downpayment, how long will this person have to wait before the savings account contains this amount?

$$A = 2400, \quad P = 1800, \quad i = 0.02$$

n represents the number of quarter year intervals.

$$A = P(1+i)^n$$
$$2400 = 1800(1.02)^n$$
$$24 = 18(1.02)^n$$
$$\log 24 = \log\left[18(1.02)^n\right]$$
$$\log 24 = \log 18 + n\log 1.02$$
$$n = \frac{\log 24 - \log 18}{\log 1.02}$$
$$= 14.527 \text{ quarter year periods}$$

Thus, 15 quarter year intervals are required since no interest will be paid for a fraction of a quarter year.

This person has to wait $\frac{15}{4} = 3\frac{3}{4}$ years .

d. Determine the amount of money that this person should invest today at 12% per annum compounded semiannually in order to produce $5200 in 6 years.

$$i = \frac{12}{2}$$
$$= 6\%$$
$$= 0.06$$
$$A = \$5200$$
$$n = 12 \text{ interest periods}$$
$$P_v = A(1+i)^{-n}$$
$$= 5200(1.06)^{-12}$$
$$= \$2584.24$$

$2584.24 should be invested now, at 12% per annum compounded semiannually in order to produce $5200 in 6 years.

Topic 3

_____ marks

Topic 4 : Series

③ 1. Write the series $\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \ldots + 32$ in summation notation.

$$a = \frac{1}{8},$$

$$r = \frac{t_2}{t_1}$$

$$= \frac{\frac{1}{4}}{\frac{1}{8}}$$

$$= 2$$

$$t_n = ar^{n-1}$$

$$t_n = \left(\frac{1}{8}\right)(2)^{n-1} \text{ or } 2^{n-4}$$

The number of terms must be determined.

$$32 = \left(\frac{1}{8}\right)(2)^{n-1}$$

$$256 = 2^{n-1}$$

$$2^8 = 2^{n-1}$$

$$n = 9$$

The summation notation is $\displaystyle\sum_{n=1}^{9} \frac{1}{8}(2)^{n-1}$ or $\displaystyle\sum_{n=1}^{9}(2)^{n-4}$.

② 2. Expand the series $\displaystyle\sum_{n=3}^{8} n^2 - 8n$; then find the sum of the series.

If $n = 3$, then $n^2 - 8n = 3^2 - 8(3) = -15$.

If $n = 4$, then $n^2 - 8n = 4^2 - 8(4) = -16$.

If $n = 8$, then $n^2 - 8n = 8^2 - 8(8) = 0$.

$-15 + (-16) + (-15) + (-12) + (-7) + 0 = -65$

Topic 4

_____ marks

Topic 5 : Arithmetic Series

③ 1. Determine the sum of the first ten terms of the arithmetic series $26 + 22 + 18 + \ldots$.

$$a = 26, \ d = -4, \ n = 10$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2(26) + (10-1)(-4)]$$

$$= 5[52 - 36]$$

$$S_{10} = 80$$

④ 2. Determine the sum of the arithmetic series $-30 + (-24) + (-18) + \ldots + 84$.

The number of terms, n, must be determined.

$$a = -30, \ d = 6, \ t_n = 84$$

$$t_n = a + (n-1)d$$

$$84 = -30 + (n-1)(6)$$

$$84 = -30 + 6n - 6$$

$$120 = 6n$$

$$n = 20$$

$$S_n = \frac{n}{2}(a + t_n)$$

$$= \frac{20}{2}(-30 + 84)$$

$$= 10(54)$$

$$S_n = 540$$

(5) 3. The sum of the first 16 terms of an arithmetic series is -80 and the sum of the first 22 terms of the same series is -506. Determine the common difference, the first term, and the sum of the first 12 terms.

$S_{16} = -80, n = 16$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$-80 = \frac{16}{2}[2a + (16-1)d]$

$-80 = 8[2a + 15d]$

$-10 = 2a + 15d$ ①

$S_{22} = -506, n = 22$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$-506 = \frac{22}{2}[2a + (22-1)d]$

$-506 = 11[2a + 21d]$

$-46 = 2a + 21d$ ②

$-10 = 2a + 15d$ ①

$-46 = 2a + 21d$ ②

Subtract ① from ②.

$-36 = 6d$

$d = -6$

Substitute $d = -6$ in ①.

$-10 = 2a + 15d$

$-10 = 2a + 15(-6)$

$80 = 2a$

$a = 40$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2(40) + (11)(-6)]$

$= 6(80 - 66)$

$= 6(14)$

$S_{12} = 84$

④ 4. Determine the number of terms in the arithmetic series $34 + 26 + 18 + \ldots$ which have a sum of -330.

$$a = 34, \quad d = -8$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$-330 = \frac{n}{2}\left[2(34) + (n-1)(-8)\right]$$

$$-330 = \frac{n}{2}\left[68 - 8n + 8\right]$$

$$-330 = \frac{n}{2}\left[76 - 8n\right]$$

$$-660 = 76n - 8n^2$$

$$8n^2 - 76n - 660 = 0$$

$$2n^2 - 19n - 165 = 0$$

$$(2n + 11)(n - 15) = 0$$

$$n = \frac{-11}{2} \qquad n = 15$$

↑

Not acceptable; n must be a whole number.

15 terms give a sum of -330.

④   5. A person owes $1700 to a friend. This person makes monthly payments to his friend. The first payment is $80 and every other payment is $20 more than the previous month. How many months does it take this person to pay off the $1700?

$$a = 80, \ d = 20, \ S_n = 1700$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$1700 = \frac{n}{2}\left[2(80) + (n-1)(20)\right]$$

$$1700 = \frac{n}{2}(160 + 20n - 20)$$

$$1700 = \frac{n}{2}(140 + 20n)$$

$$3400 = 140n + 20n^2$$

$$20n^2 + 140n - 3400 = 0$$

$$n^2 + 7n - 170 = 0$$

$$(n + 17)(n - 10) = 0$$

$$n = -17 \qquad n = 10$$

$$\uparrow$$

Not acceptable; n must be a whole number.

It takes 10 months for this person to pay off the $1700.

Topic 5

_____ marks

Topic 6 : Geometric Series

④ 1. Determine the sum of the geometric series $135 + 90 + 60 + \ldots + \frac{320}{27}$.

$$a = 135, \quad r = \frac{2}{3}, \quad t_n = \frac{320}{27}$$

The value of n must be determined using $t_n = ar^{n-1}$.

$$\frac{320}{27} = 135\left(\frac{2}{3}\right)^{n-1}$$

$$\frac{320}{27} \times \frac{1}{135} = \left(\frac{2}{3}\right)^{n-1}$$

$$\frac{64}{729} = \left(\frac{2}{3}\right)^{n-1} \qquad\qquad \left(\frac{2}{3}\right)^6 = \frac{64}{729}$$

$$\left(\frac{2}{3}\right)^6 = \left(\frac{2}{3}\right)^{n-1}$$

$$6 = n - 1$$

$$n = 7$$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r}$$

$$S_7 = \frac{135\left[1 - \left(\frac{2}{3}\right)^7\right]}{1 - \frac{2}{3}}$$

$$S_7 = \frac{135\left(1 - \frac{128}{2187}\right)}{\frac{1}{3}}$$

$$S_7 = 3(135)\left(\frac{2059}{2187}\right)$$

$$S_7 = 381\frac{8}{27} \qquad \text{or} \qquad = 381.\overline{296}$$

④ 2. In a geometric series, the sum of the first six terms is –168 and the sum of the first seven terms is 344. The fifth term is 128. Determine the first four terms of this series. Then calculate S_{12} for the series.

$S_6 = -168, \ S_7 = 344, \ t_5 = 128$

Thus, $S_6 + t_7 = S_7$.

$-168 + ar^6 = 344$. $ar^6 = t_7$ since $t_n = ar^{n-1}$.

$ar^6 = 512$

But $t_5 = ar^4 = 128$.

Divide ar^6 by ar^4.

$$\frac{ar^6}{ar^4} = \frac{512}{128}$$

$r^2 = 4$

$r = -2$ (2 will not work.)

$ar^6 = 512$

$a(-2)^6 = 512$

$a = 8$

The first four terms are 8, – 16, 32, and – 64.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{12} = \frac{8[(-2)^{12} - 1]}{-2 - 1}$$

$$S_{12} = \frac{8(4096 - 1)}{-3}$$

$S_{12} = -10\ 920$

④ 3. A savings account receives deposits of $500 every 6 months for 5 years. The interest rate is 10.5% per annum, compounded quarterly. Determine the total amount of money in the annuity account after the last deposit is made.

$$i = \frac{10.5}{4}$$
$$= 2.625\%$$
$$= 0.026\ 25, \quad n = 10$$
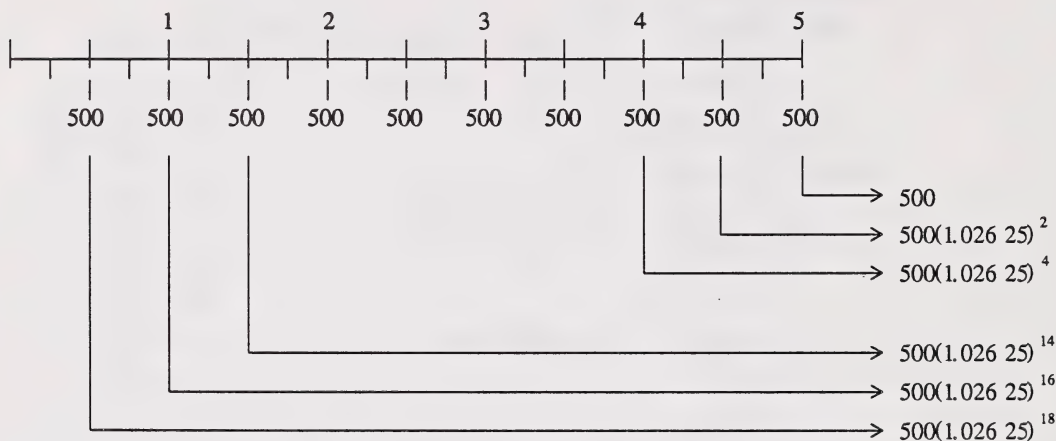
$$A = 500 + 500(1.026\ 25)^2 + 500(1.026\ 25)^4 + \ldots + 500(1.026\ 25)^{18}$$
$$r = 1.026\ 25^2$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{500\left[(1.026\ 25^2)^{10} - 1\right]}{(1.026\ 25)^2 - 1}$$

$$S_{10} = \frac{500(1.026\ 25^{20} - 1)}{1.053\ 189\ 062 - 1}$$

$$S_{10} = 6383.35$$

There will be $6383.35 in the account.

④ 4. Determine the amount of money which must be invested today in order to provide $3600 at the end of each year for 5 years. The money invested earns 9.5 % per annum, compounded semiannually.

$$i = \frac{9.5}{2}$$
$$= 4.75\%$$
$$= 0.0475$$
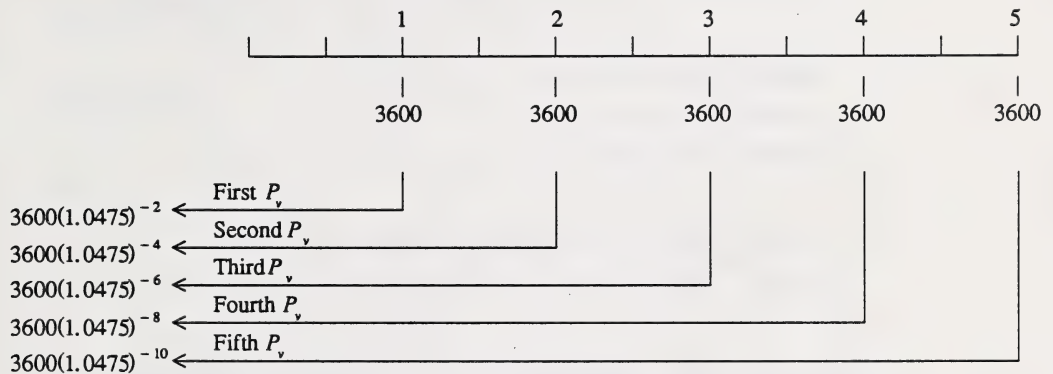
$$P_v = A(1+i)^{-n}$$

The first present value, $3600(1.0475)^{-2}$, is the money that must be invested now in order to receive $3600 one year from now. The amount of this first present value is $3280.91.

The second present value, $3600(1.0475)^{-4}$, is the money that must be invested now in order to receive $3600 two years from now. The amount of this second present value is $2990.10.

In reverse order, $P_v = 3600(1.0475)^{-10} + 3600(1.0475)^{-8} + \ldots + 3600(1.0475)^{-2}$

$a = 3600(1.0475)^{-10}, \quad r = (1.0475)^2, n = 5$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_n = \frac{3600(1.0475)^{-10}\left[(1.0475^2)^5 - 1\right]}{(1.0475)^2 - 1}$$

$$S_n = \frac{3600 - 3600(1.0475)^{-10}}{1.097\ 256\ 25 - 1}$$

$$= 13\ 743.03$$

They should invest $13 743.03.

④ 5. A county buys some road maintenance equipment for $60 000. The county wants to pay for this equipment over a period of 6 years. If the interest rate is 9% per annum, compounded annually, determine the annual payment to cover both principal and interest.
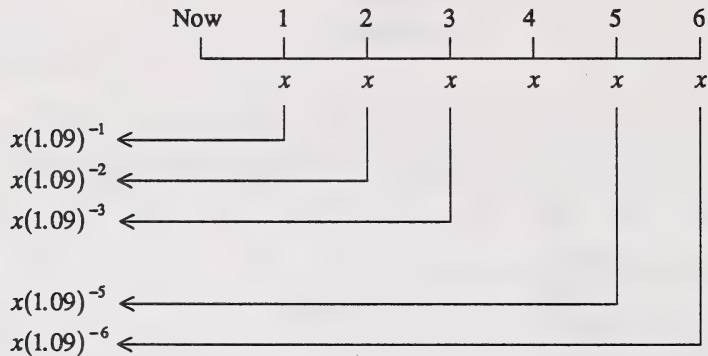
Let x be each annual payment.

$$P_v = x(1.09)^{-6} + x(1.09)^{-5} + \ldots + x(1.09)^{-2} + x(1.09)^{-1}$$

$$a = x(1.09)^{-6}, \quad r = 1.09, \quad n = 6 \quad P_v = 60\ 000$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$60\ 000 = \frac{x(1.09)^{-6}(1.09^6 - 1)}{1.09 - 1}$$

$$60\ 000 = \frac{x - x(1.09)^{-6}}{0.09}$$

$$60\ 000 = \frac{x - 0.596\ 267\ 326x}{0.09}$$

$$x = \$13\ 375.19$$

Topic 6

_____ marks

Topic 7 : Infinite Series and Limits

②

1. Determine $\lim\limits_{n\to\infty}\left(\dfrac{3n}{n-2}+\dfrac{4n^2-1}{n^2+2}\right)$.

$$\lim_{n\to\infty}\left[\frac{\frac{3n}{n}}{\frac{n}{n}-\frac{2}{n}}+\frac{\frac{4n^2}{n^2}-\frac{1}{n^2}}{\frac{n^2}{n^2}+\frac{2}{n^2}}\right]$$

$$=\frac{3}{1-0}+\frac{4-0}{1+0}$$

$$=3+4$$

$$=7$$

③

2. Express the number $2.139\ 393\ 9\ldots$ as a rational number in lowest terms.

$2.139\ 393\ 9\ldots\ =2.1+0.039+0.000\ 39+0.000\ 003\ 9+\ \ldots$

The value 2.1 does not belong to the geometric series $0.039 + 0.000\ 39 + \ldots$.
For the geometric series,

$$a = 0.039 \text{ and } r=\frac{t_2}{t_1}=\frac{0.000\ 39}{0.039}=0.01.$$

$$2.139\ 393\ 9\ldots\ =2.1+\frac{a}{1-r}$$

$$=2.1+\frac{0.039}{1-0.01}$$

$$=2.1+\frac{39}{990}$$

$$=\frac{21}{10}+\frac{39}{990}$$

$$=\frac{2079+39}{990}$$

$$=\frac{2118}{990}$$

$$=\frac{706}{330}$$

$$=\frac{353}{165}$$

③ 3. A rubber ball is dropped 15 m and bounces to $\frac{2}{3}$ the previous vertical distance. What total vertical distance has the ball travelled when it stops bouncing?
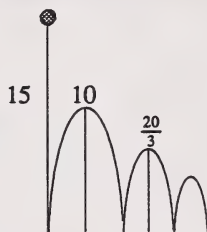
$$\text{Distance} = 15 + 2\left(\frac{a}{1-r}\right)$$

$$= 15 + 2\left(\frac{10}{1-\frac{2}{3}}\right)$$

$$= 15 + 2\left(\frac{10}{\frac{1}{3}}\right)$$

$$= 15 + 2(30)$$

$$= 15 + 60$$

$$= 75$$

The total distance travelled is 75 m.

② 4. Determine the sum of the following series: $92 + 46 + 24 + 18 + 13.5 + \ldots$.

The 92 and 46 do not belong to the geometric series. The sum must be determined in two parts.

$$\text{Sum} = (92 + 46) + \frac{a}{1-r}$$

$$= 138 + \frac{24}{1-\frac{3}{4}}$$

$$S = 138 + 96$$

$$S = 234$$

Topic 7

_____ marks
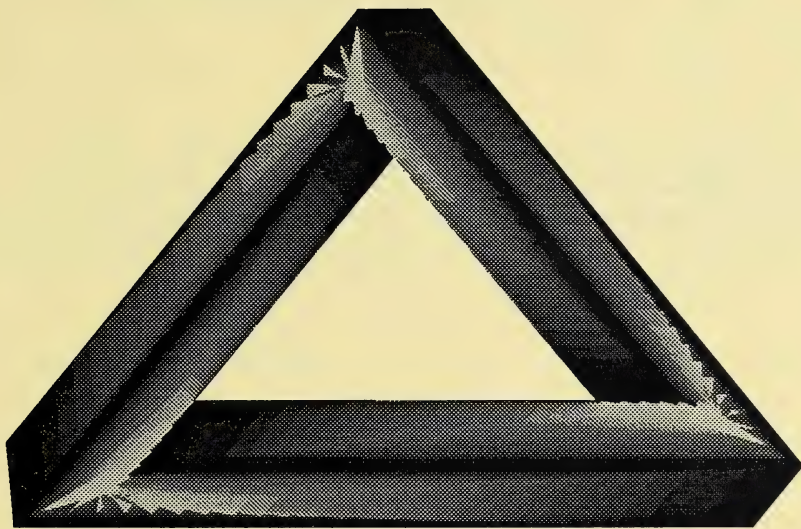
MATHEMATICS 30

LEARNING FACILITATOR'S MANUAL

UNIT 4

Distance Learning

Alberta
EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Topic 1 : Circular Paths

②

1. On the grid at the right, locate a point P with a horizontal
    coordinate of 1.1 and a vertical coordinate of 1.1.
    Calculate, to at least four decimal places, the value of

    a. sin A
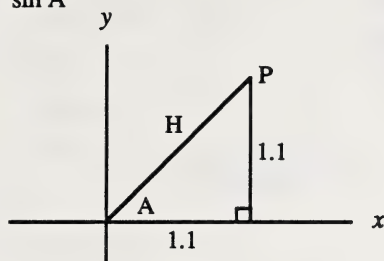
    By the Pythagorean theorem:

    $$H^2 = S^2 + S^2$$

    $$H^2 = (1.1)^2 + (1.1)^2$$

    $$H^2 = 1.21 + 1.21$$

    $$H^2 = 2.42$$

    $$H = \sqrt{2.42}$$

    $$= 1.5556$$

    Method 1: $\sin A = \dfrac{1.1}{1.5556} \left(\dfrac{\text{side opposite}}{\text{hypotenuse}} \right)$

    $$= 0.7071$$

    Method 2: Since the triangle is
    isosceles (two sides
    are equal),
    $\angle A = 45°.$
    $\therefore \sin 45° = 0.7071$

    b. sec A
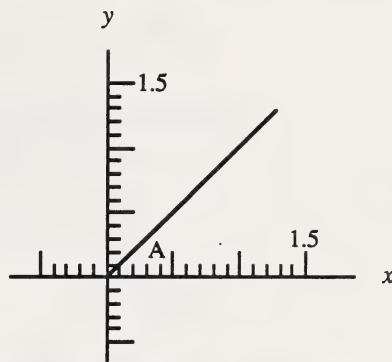
    Method 1: $\sec A = \dfrac{1}{\cos A}$

    $\cos A = \dfrac{1.1}{1.5556} \left(\dfrac{\text{side adjacent}}{\text{hypotenuse}} \right)$

    $$= 0.7071$$

    $\sec A = \dfrac{1}{0.7071}$

    $$= 1.4142$$

    Method 2: $\sec 45° = \dfrac{1}{\cos 45°}$

    $$= \dfrac{1}{0.7071}$$

    $$= 1.4142$$

② 2. In a right angle triangle, $\tan A = \dfrac{12.23}{5.46}$. Calculate, correct to three decimal places, the value of

    a.   sin A

$$\tan A = \frac{12.23}{5.46}$$
$$\tan A = 2.2399$$
$$\angle A = 65.942°$$
$$\sin 65.942° = 0.913$$

    b.   cos A

$$\cos 65.942° = 0.408$$

① 3. Rewrite the function tan 302° in terms of its reference angle.

    tan 302° = − tan _____°

    $\tan 302° = \tan(360° − 58°)$
              $= − \tan 58°$ (tan is negative in the 4th quadrant.)

② 4. If $\cos \theta = -\dfrac{1}{4}$ and $\tan \theta$ is positive, find $\csc \theta$. Write the answer in exact value.

By the Pythagorean formula:

$$AC^2 = AB^2 + BC^2$$

$$4^2 = AB^2 + (-1)^2$$

$$AB^2 = 16 - 1$$

$$AB^2 = 15$$

$$AB = -\sqrt{15} \quad \text{(negative since in Quadrant 3)}$$

$$\csc \theta = \frac{AC}{AB}$$

$$= \frac{4}{-\sqrt{15}}$$

$$= \frac{-4\sqrt{15}}{15}$$

cos negative and tan positive is a third quadrant angle.
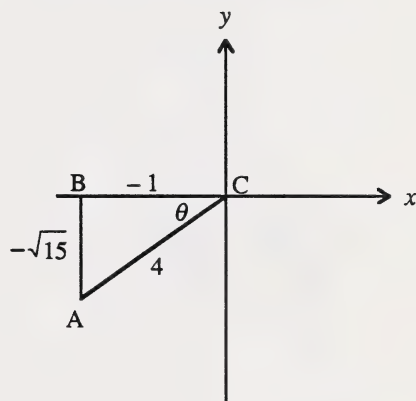
② 5. A smokestack 24.6 m tall casts a shadow 23.9 m long. What is the angle of elevation of the sun?

$$\tan \theta = \frac{24.6}{23.9}$$

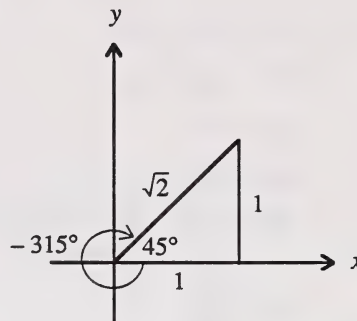$$\tan \theta \doteq 1.0293$$
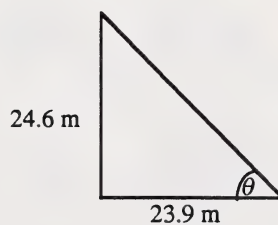
$$\theta \doteq 45.83°$$

The sun's elevation is 45.83°.

24.6 m

23.9 m

③

6. Find the value of each using the special triangles. Draw a diagram to see in which quadrant the terminal arm lies.

a. $\sec(-315°) = \sec 45°$ (This is the reference angle between the x-axis and the terminal arm.)

$$\sec 45° = \frac{\sqrt{2}}{1}$$

$$= \sqrt{2}$$

b. $3\sin(-150°) - 4\tan 300°$

$\sin(-150°) = -\sin 30°$. This reference angle is in the 3rd quadrant. Therefore, sine is negative.

$\tan 300°$ is in the 4th quadrant. The reference angle is 60° and tangent is negative. Thus, $\tan 300° = -\tan 60°$.
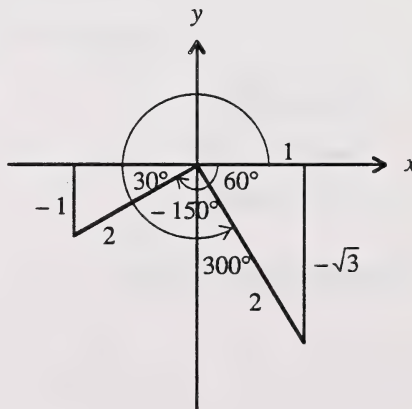
$$3\sin(-150°) - 4\tan 300°$$

$$= 3\left(-\frac{1}{2}\right) - 4\left(-\sqrt{3}\right)$$

$$= -\frac{3}{2} + 4\sqrt{3}$$

$$= -1.5 + 4\sqrt{3}$$

$$= 5.4282$$

4

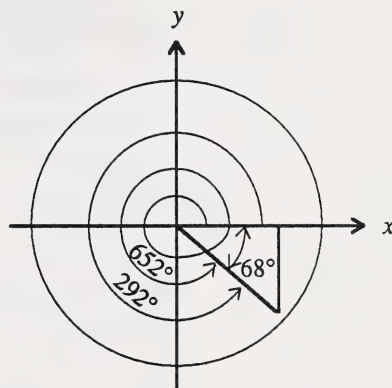③ 7. State the quadrant in which each angle terminates and find the measure of the reference angle.

a. 652°

Quadrant 4
The angle between 0° and 360° coterminal with 652°
is 652° − 360° = 292°.

$$R = 360° − 292°$$
$$= 68°$$

b. −710°

Quadrant 1
Angle between 0° and 360° coterminal with −710°
is −710° + 2(360°).
−710° + 720° = 10°
$$R = 10°$$
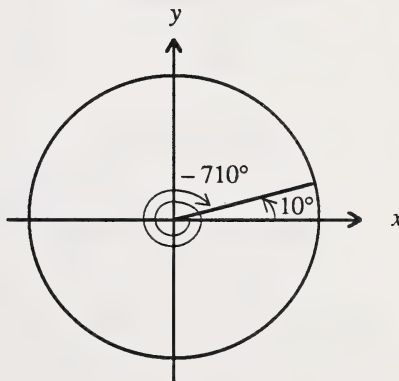
Topic 1

_____ marks

Topic 2 : The Unit Circle

① 1. Point P is on the unit circle. The coordinates of P are (0.438, 0.899). Determine the value of A, where A is the angle made by the positive x-axis and the line passing through the point P. Give the answer to the nearest whole number.

$$\tan A = \frac{0.899}{0.438}$$

$$= 2.0525$$

$$\angle A \doteq 64.0243°$$

$$\angle A \doteq 64°$$

or $\sin A = 0.899$

$$\angle A \doteq 64°$$

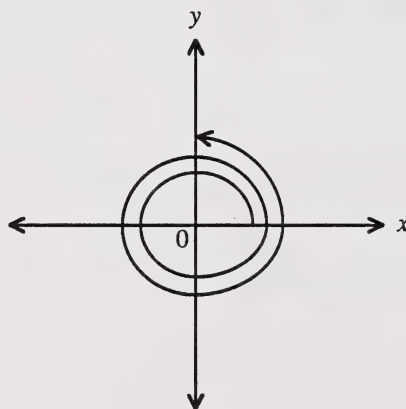or $\cos A = 0.438$

$$\angle A \doteq 64°$$
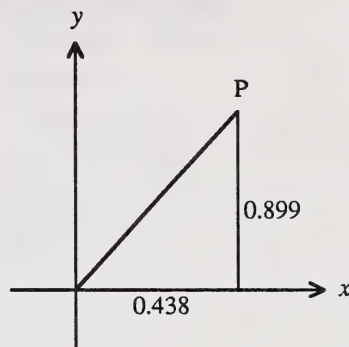
① 2. Refer to the figure given below.

What is the measure of the given angle in radians?

$$\text{Angle} = \frac{9}{2}\pi \text{ radians}$$

(2) 3. Express the following in degrees.

   a. 2.6π b. 4.2π

$$2.6\pi = (2.6\pi)\left(\frac{180°}{\pi}\right) \qquad\qquad 4.2\pi = (4.2\pi)\left(\frac{180°}{\pi}\right)$$

$$= (2.6)(180°) \qquad\qquad\qquad\quad = (4.2)(180°)$$

$$= 468° \qquad\qquad\qquad\qquad\quad = 756°$$

(1) 4. Calculate sin 0.1 radians to four decimal places.

$$1 \text{ radian} = 57.3° \quad \left(\frac{180°}{\pi} \doteq 57.3°\right)$$
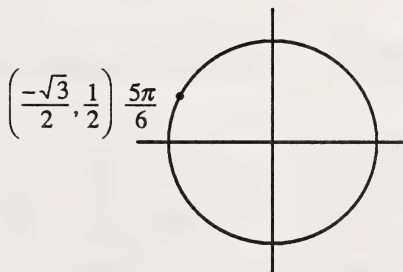
$$0.1 \text{ radian} = 5.73°$$

$$\sin 5.73° = 0.0998$$

(2) 5. What is the exact value of $\tan \dfrac{5\pi}{6}$?

$$= \frac{\dfrac{1}{2}}{\dfrac{-\sqrt{3}}{2}}$$

$$= \frac{1}{-\sqrt{3}}$$

$$= \frac{1}{-\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{-\sqrt{3}}{3}$$

$$\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right) \ \frac{5\pi}{6}$$

② 6. The spoke of a bicycle wheel is 32 cm long. What is the distance travelled by a point on the rim of the wheel if the radian measure is 1.9?

$$\theta = \frac{a}{r}$$

$$1.9 = \frac{a}{32}$$

$$a = 60.8$$

The distance travelled by the point on the rim of the wheel is 60.8 cm.

③ 7. If $\cos \theta = -\frac{3}{4}$, $90° < \theta < 180°$, determine the exact values of $\tan \theta$ and $\csc \theta$.

Method 1:

$$x^2 + y^2 = 1$$

$$\left(-\frac{3}{4}\right)^2 + y^2 = 1$$

$$\frac{9}{16} + y^2 = 1$$

$$y^2 = 1 - \frac{9}{16}$$

$$y^2 = \frac{7}{16}$$

$$y = \frac{\sqrt{7}}{4}$$

$$\tan \theta = \frac{y}{x}$$

$$= \frac{\frac{\sqrt{7}}{4}}{-\frac{3}{4}}$$

$$= \frac{-\sqrt{7}}{3}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$= \frac{1}{\frac{\sqrt{7}}{4}}$$

$$= \frac{4}{\sqrt{7}}$$

$$= \frac{4\sqrt{7}}{7}$$

Method 2: Since $\cos \theta$ is negative, it is a second quadrant angle.
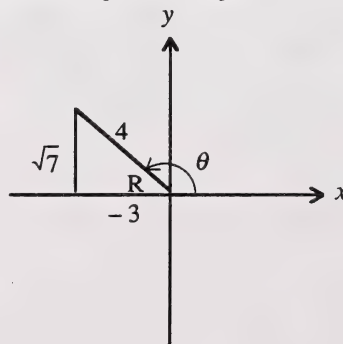
$$4^2 = y^2 + (-3)^2$$

$$16 = y^2 + 9$$

$$y^2 = 7$$

$$y = \sqrt{7}$$

$$\tan \theta = \frac{\sqrt{7}}{-3}$$

$$\csc \theta = \frac{4}{\sqrt{7}}$$
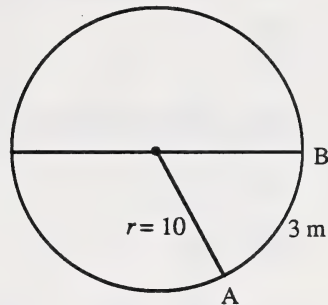
$$= \frac{4\sqrt{7}}{7}$$

② 8. A person on a ferris wheel moved a distance of 3 m from position A to position B. If the diameter of the wheel is 20 m, find the measure of the central angle in radians and degrees.

$\theta = \dfrac{a}{r}$

$\theta = \dfrac{3}{10}$

$\theta = 0.3$ radian

$0.3 \text{ radian} = 0.3 \text{ rad} \times \dfrac{180°}{\pi \text{ rad}}$

$\qquad\qquad\quad = 17.19°$

The central angle is 0.3 radian or 17.19°.

② 9. A pendulum 45 cm long swings through an angle of 60°. What is the length of the arc through which the end of the pendulum swings?

$\theta = \dfrac{a}{r}$

$\dfrac{\pi}{3} = \dfrac{a}{45}$

$a = 45 \times \dfrac{\pi}{3}$

$\quad = 15\pi$

$\quad = 47$

$60° = 60° \times \dfrac{\pi}{180°}$

$\quad\ = \dfrac{\pi}{3}$ radians

The length of the arc is 47 cm.

Topic 2

_____ marks

Topic 3 : Solving Equations

③   1.   Solve each of the following if $0 < A < 2\pi$.

     a. $\sec A = -2$
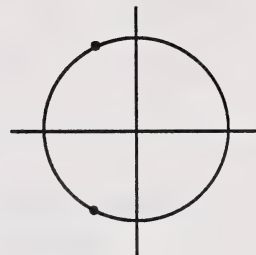
        sec is negative in Quadrants 2 and 3.

$$\cos A = \frac{1}{\sec A}$$

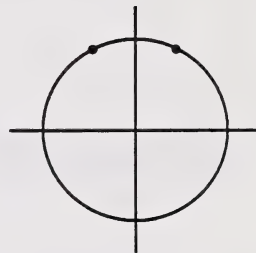$$\cos A = \frac{1}{-2}$$

$$A = \frac{2\pi}{3}, \ \frac{4\pi}{3}.$$

     b. $\sin A = \frac{\sqrt{3}}{2}$
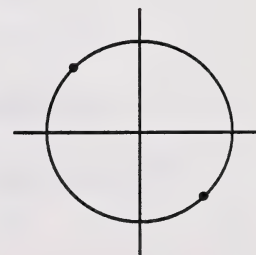
        sin is positive in Quadrants 1 and 2.

$$A = \frac{\pi}{3}, \ \frac{2\pi}{3}$$

     c. $\tan A = -1$

        tan is negative in Quadrants 2 and 4.

$$A = \frac{3\pi}{4}, \ \frac{7\pi}{4}$$

③ 2. a. Solve the following equation for θ, given that $0° \le \theta < 360°$.

$8 \sin \theta \tan \theta - 4 \tan \theta = 0$

Factorize. $4 \tan \theta$ is common.

$4 \tan \theta (2 \sin \theta - 1) = 0$

$4 \tan \theta = 0$	$2 \sin \theta - 1 = 0$
$\tan \theta = 0$	$2 \sin \theta = 1$
$\theta = 0, \ \pi$	$\sin \theta = \dfrac{1}{2}$
$= 0°, \ 180°$	$\theta = \dfrac{\pi}{6}, \ \dfrac{5\pi}{6}$
	$= 30°, \ 150°$

b. How many angles satisfy this equation?

4 angles

c. What is the largest quadrantal angle that satisfies this equation?

180°

③ 3. Solve for x if $\dfrac{\pi}{2} \le x < \pi$.

$2 \sin x \cos x = \cos x$

$2 \sin x \cos x - \cos x = 0$

Factorize. $\cos x$ is common.

$\cos x (2 \sin x - 1) = 0$

$\cos x = 0$ $2 \sin x - 1 = 0$

$x = \dfrac{\pi}{2}$ $2 \sin x = 1$

$\sin x = \dfrac{1}{2}$

$x = \dfrac{5\pi}{6}$

(3)

4. a. Solve the following equation for y, given that $0° \leq y < 360°$.

$$16 \tan^2 y = 1 - 33 \tan^2 y$$
$$16 \tan^2 y + 33 \tan^2 y = 1$$
$$49 \tan^2 y = 1$$
$$\tan^2 y = \frac{1}{49}$$
$$\tan y = \pm \frac{1}{7}$$
$$y = 8°, \ 188°, \ 172°, \ 352°$$

b. How many angles satisfy this equation?

4 angles

c. What third quadrant angle satisfies this equation?

188°

d. What fourth quadrant angle satisfies this equation?

352°

(3)

5. Solve the equation $2 \cos^2 \theta - 3 \cos \theta + 1 = 0$ for $0° \leq \theta \leq 360°$.

Factorize. Use the trinomial method.

$$(2 \cos \theta - 1)(\cos \theta - 1) = 0$$

$2 \cos \theta - 1 = 0$	$\cos \theta - 1 = 0$
$2 \cos \theta = 1$	$\cos \theta = 1$
$\cos \theta = \frac{1}{2}$	$\theta = 0°, \ 360°$
$\theta = 60°, \ 300°$	

Topic 3

_____ marks

Topic 4 : Graphs of Trigonometric Functions

③ 1. Sketch the graph of $y = -\cos\theta, -\pi \le \theta \le 2\pi$.
Give the domain and range.

θ	$-\pi$	$\frac{-\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = -\cos\theta$	1	0	-1	0	1	0	-1
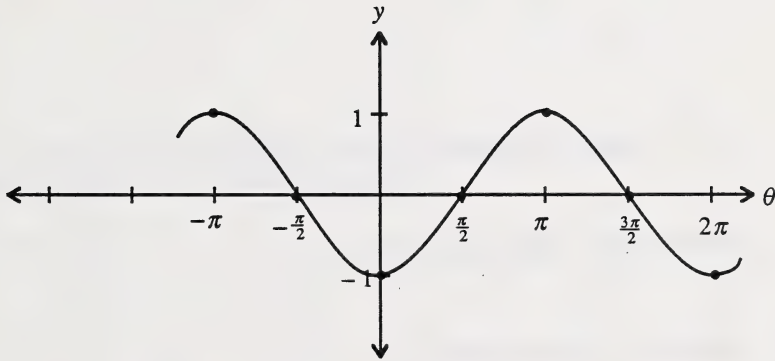
Domain $= -\pi \le \theta \le 2\pi$

Range $= -1 \le y \le 1$

① 2. Give the domain and period for $\tan\theta$.

Domain: $\left\{\theta \middle| \theta \ne \frac{\pi}{2} + n\pi, n \in I\right\}$

Period: π

(1) 3. For $y = \csc\ \theta$, state the following:

    a. domain $\{\theta \mid \theta \neq n\pi, n \in I\}$

    b. period 2π

    c. range $\{y \mid y \leq -1 \text{ or } y \geq 1\}$

(1) 4. Give the domain, range, and period for cot θ.

    Domain: $\{\theta \mid \theta \neq n\pi, n \in I\}$

    Range: $\{y \mid y \in R\}$

    Period: π

Topic 4

_____ marks

Topic 5 : Amplitude, Period, Phase Shift, and Vertical Translation

1. Which of the following is the period of the function $y = \cos 2\theta$?
    Circle the letter which matches the correct response.

    (A.) π $\qquad$ Period $= \dfrac{2\pi}{b}$, where b equals the constant in $y = \cos b\theta$.

    B. 2

    C. 3π $\qquad$ $\dfrac{2\pi}{2} = \pi$

    D. 0

2. Write an equation for the sine function having the following properties:
    amplitude 3, period π, phase shift $\dfrac{\pi}{4}$ to the right

    $y = 3 \sin 2\left(\theta - \tfrac{\pi}{4}\right)$

3. If $f(x) = -5 \cos\left(\tfrac{1}{2}\theta + \tfrac{\pi}{4}\right) + 3$, state the amplitude, period, phase shift, and vertical translation.

    $f(x) = -5 \cos \tfrac{1}{2}(\theta + \tfrac{\pi}{2}) + 3$

    amplitude: $|-5| = 5$

    period: $\dfrac{2\pi}{\tfrac{1}{2}} = 4\pi$

    phase shift: $\dfrac{\pi}{2}$ to the left

    vertical translation: 3

④

4. Sketch the curve of $y = 2\sin\left(\frac{1}{2}\theta + \frac{\pi}{8}\right) + 1$.

Give the range.

$y = 2\sin\left(\frac{1}{2}\theta + \frac{\pi}{8}\right) + 1$ becomes $y = 2\sin\frac{1}{2}\left(\theta + \frac{\pi}{4}\right) + 1$.

$y = 2\sin\frac{1}{2}\theta$

This page has been provided for your calculations.

$y = 2\sin(\tfrac{1}{2}\theta + \tfrac{\pi}{8}) + 1$

$y = 2\sin\tfrac{1}{2}(\theta + \tfrac{\pi}{4})$
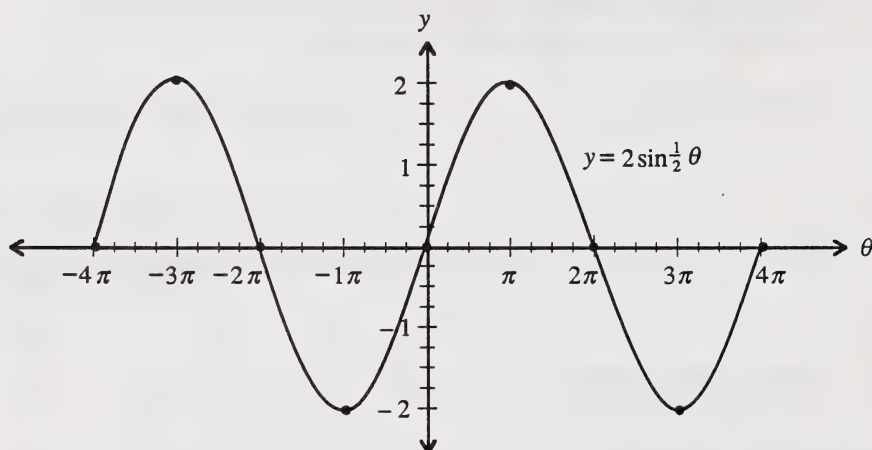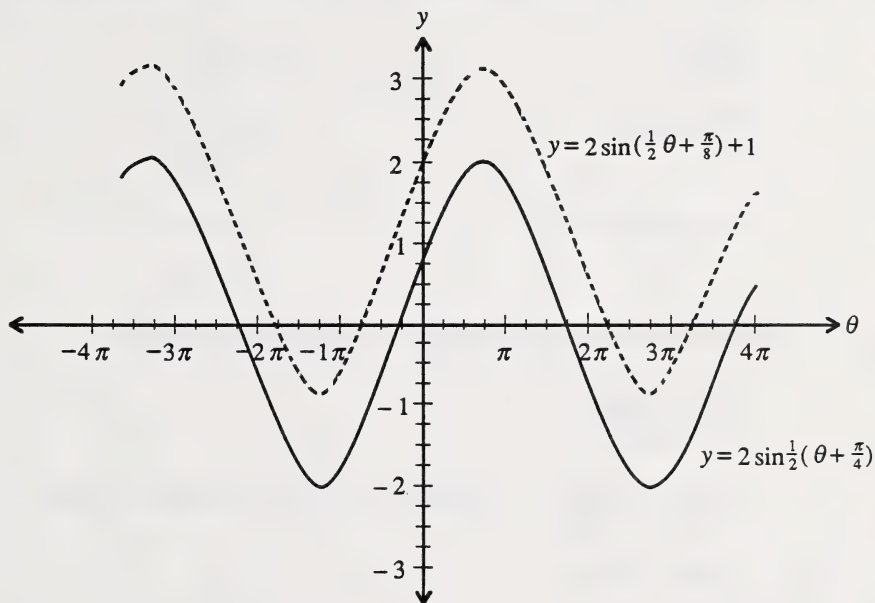
Range: $\{y \mid -1 \le y \le 3\}$

Topic 5

_____ marks

Topic 6 : Trigonometric Identities

①   1. Simplify the following expression by using the reciprocal relation.

$$\frac{\cos y}{\sec y}$$

$$\cos y = \frac{1}{\sec y}$$

$$\therefore \cos y \cos y = \cos^2 y$$

②   2. Simplify the following expression by using the quotient relations.

$$\frac{\cos y - \sin^2 y}{\sin y}$$

$$= \frac{\cos y}{\sin y} - \frac{\sin^2 y}{\sin y} \qquad \text{(Divide each term in the numerator by } \sin y.)$$

$$= \cot y - \sin y$$

④   3. Prove that $\cos \theta + \tan \theta \sin \theta = \sec \theta$.

$$\cos \theta + \tan \theta \sin \theta = \cos \theta + \frac{\sin \theta}{\cos \theta} \sin \theta$$

$$= \cos \theta + \frac{\sin^2 \theta}{\cos \theta}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta}$$

$$= \sec \theta$$

$$= RS$$

③

4. Prove that $\left(1+\cot^2\theta\right)\cos^2\theta = \cot^2\theta.$

$\left(1+\cot^2\theta\right)\cos^2\theta = \csc^2\theta\cos^2\theta$ $\qquad$ $\left(1+\cot^2\theta = \csc^2\theta\right)$

$\qquad\qquad = \dfrac{1}{\sin^2\theta}\cos^2\theta$

$\qquad\qquad = \dfrac{\cos^2\theta}{\sin^2\theta}$

$\qquad\qquad = \cot^2\theta$
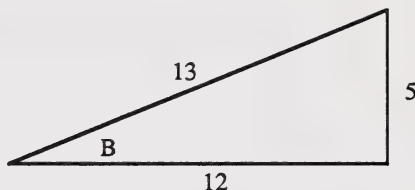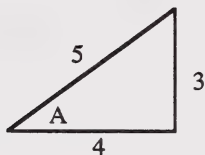
$\qquad\qquad = RS$

Topic 6

_____ marks

Topic 7 : Sum Formulas

① 1. Simplify $\cot\left(-\frac{\pi}{2}+2k\right)$ by writing it in terms of a single trigonometric function using $2k$ where $0° < k < 45°$.

$\cot\left(\frac{-\pi}{2}+2k\right)$ is in the fourth quadrant. Therefore, k must be positive.

$\therefore \cot\left(\frac{-\pi}{2}+2k\right) = -\tan 2k.$

③ 2. If $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$, and A and B are acute angles, find $\sin(A+B)$.

$$\sin A = \frac{3}{5},\ \cos A = \frac{4}{5}$$

$$\sin B = \frac{5}{13},\ \cos B = \frac{12}{13}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \left(\frac{3}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{5}{13}\right)$$

$$= \frac{36}{65} + \frac{20}{65}$$

$$= \frac{56}{65}$$

$$\doteq 0.8615$$

④ 3. Evaluate tan 15° using the trigonometric subtraction formula.

$$\tan 15° = \tan(60° - 45°)$$

$$= \frac{\sin(60° - 45°)}{\cos(60° - 45°)}$$

$$= \frac{\sin 60° \cos 45° - \cos 60° \sin 45°}{\cos 60° \cos 45° + \sin 60° \sin 45°}$$

$$= \frac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}}{\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}}$$

$$= \frac{\frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3}$$

Topic 7

_____ marks

Topic 8 : Sine and Cosine Laws

⑤

1. A surveyor marked off a 164 m baseline MN on one side of a river. A tree on the other side of the river was noted as a landmark. From M, the angle formed between the line of sight to the landmark and the baseline was measured to be 45°. From N, the angle formed between the line of sight to the landmark and the baseline was measured to be 77 °. Determine the distance from M to the tree.
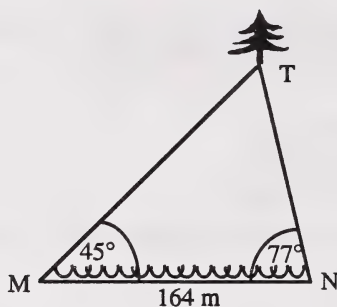
$$\angle MTN = 180° - 45° - 77°$$

$$= 58°$$

$$\frac{n}{\sin N} = \frac{t}{\sin T}$$

$$\frac{n}{\sin 77°} = \frac{164}{\sin 58°}$$

$$n = \frac{164 \sin 77°}{\sin 58°}$$

$$n \doteq 188 \text{ m}$$

The distance from M to the tree is 188 m.

(5)   2. The distances between three cities, A, B, and C, are as follows:

The distance from A to B is 330 km.
The distance from B to C is 250 km.
The distance from C to A is 480 km.

If B lies due north of A and C is in an easterly direction from B, calculate the size of angle A. (Note: The triangle is not a right triangle.) Give angle A to the nearest degree.
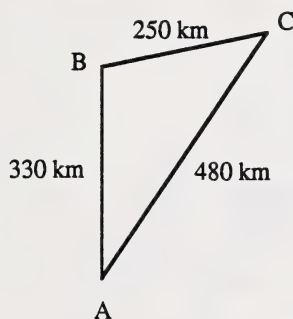
Calculate ∠BAC.

By the cosine rule,

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$250^2 = 480^2 + 330^2 - 2(480)(330) \cos A$$

$$\cos A = \frac{480^2 + 330^2 - 250^2}{2(480)(330)}$$

$$= \frac{230\,400 + 108\,900 - 62\,500}{316\,800}$$

$$= \frac{276\,800}{316\,800}$$

$$= 0.8737$$

$$\angle A = 29°$$

⑤　3.　Solve the oblique triangle ABC, given $b = 51.75$ cm, $a = 65$ cm, and B = 37.0°. For this question you will have to check for the ambiguous case. If you find the ambiguous case exists, solve for the following:

Larger angle A = _____ degrees.
Smaller angle C = _____ degrees.
Smaller side c = _____ cm.

If the ambiguous case does not exist, solve for the following:

Angle A = _____ degrees.
Angle C = _____ degrees.
Side c = _____ cm.
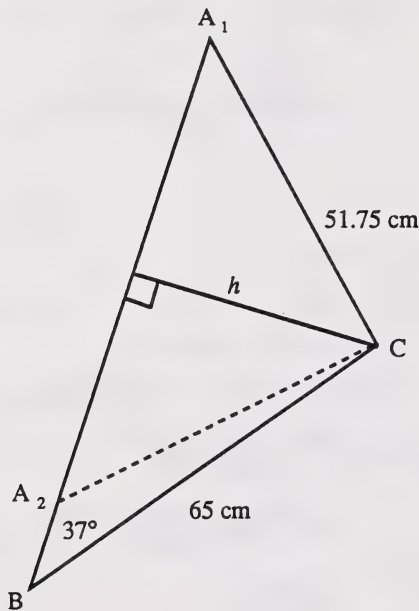
∠B < 45°

Adjacent side, BC, is the longer side.

A_1C, the opposite side to ∠B, is the shorter side.

Therefore, this is an ambiguous case.

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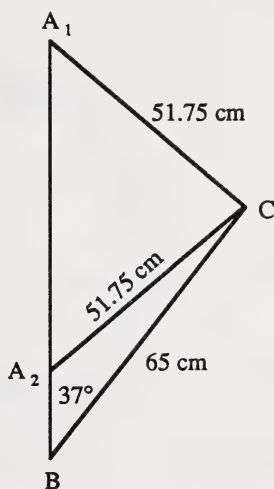
$$\frac{65}{\sin \angle BA_2C} = \frac{51.75}{\sin 37°}$$

$$\sin \angle BA_2C = \frac{65 \sin 37°}{51.75}$$

$$\sin \angle BA_2C = 0.7559$$

$$\angle BA_2C > 90°; \text{ thus, } \angle BA_2C = 180° - 49°$$

$$\text{Larger angle } A = 131°$$

$$\angle BCA_2 = 180° - 131° - 37°$$

$$\text{Smaller angle } C = 12°$$

$$\frac{BA_2}{\sin 12°} = \frac{51.75}{\sin 37°}$$

$$BA_2 = \frac{51.75 \sin 12°}{\sin 37°}$$

$$BA_2 \doteq 17.88 \text{ cm}$$

$$\text{Smaller side } c \doteq 17.88 \text{ cm}$$

⑤

4. A side of a regular nonagon measures 22 cm. Determine, to the nearest whole number, the diameter of the circle and the area of the inscribed nonagon.
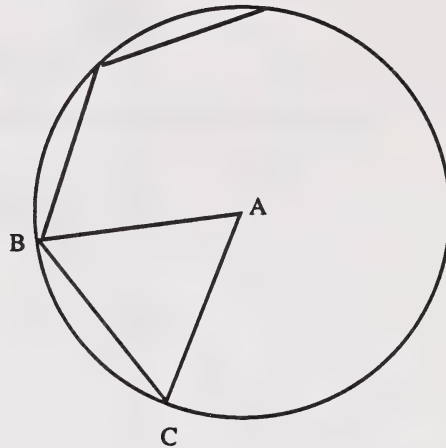
The measure of a central angle

$$= \frac{360°}{9}$$

$$= 40°.$$

Each angle opposite the congruent sides of the isosceles $\triangle$ is

$$\frac{180° - 40°}{2}$$

$$= \frac{140°}{2}$$

$$= 70°.$$

This page has been provided for your calculations.

By the Sine Rule,

$$\frac{AB}{\sin C} = \frac{BC}{\sin A}$$

$$\frac{AB}{\sin 70°} = \frac{22}{\sin 40°}$$

$$AB = \frac{22 \sin 70°}{\sin 40°}$$

$$AB = 32$$

The diameter of the circle is 2×32 cm = 64 cm.

$$\text{Area} = \frac{nr^2}{2} \sin \frac{360°}{n}$$

If $r = 32$, $\quad \text{area} = \frac{9(32)^2}{2} \sin \frac{360°}{9}$

$$= 4608 \sin 40°$$

$$\text{Area} \doteq 2962 \text{ cm}^2$$

The area of the nonagon is 2962 cm^2.

If $r = 32.161\,848\,4$,

$$\text{area} = \frac{9(32.161\,848\,4)^2}{2} \sin \frac{360°}{9}$$

$$\doteq 2992$$

The area of the nonagon is 2992 cm^2.

Topic 8

_____ marks
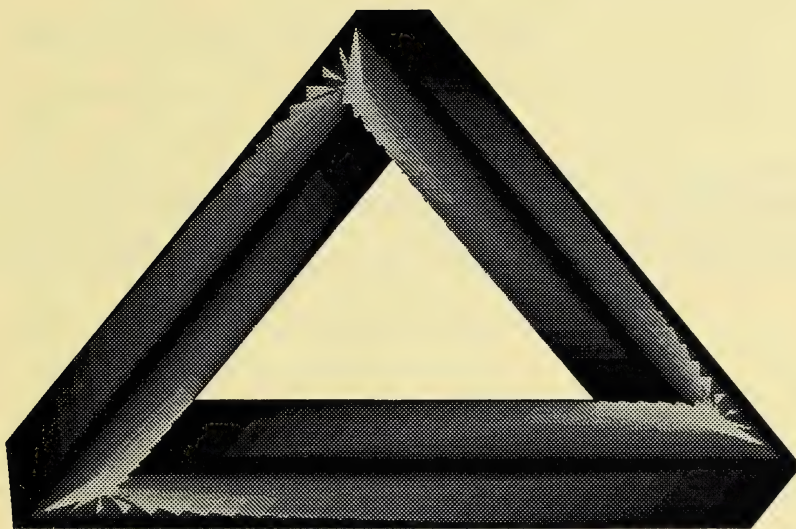
MATHEMATICS 30

LEARNING FACILITATOR'S MANUAL

UNIT 5

Distance Learning

Alberta EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Topic 1: Introduction to the Circle

② 1. A circle is centred at $(2, -3)$ and has a radius of 5.

    a. Determine the equation of the circle in standard form.

       The standard form is $(x-h)^2+(y-k)^2=r^2$, where the centre is at (h,k).

       Thus, $(h, k) = (2, -3)$.

$$(x-2)^2+\left[(y-(-3))\right]^2=5^2 \quad \text{(standard form)}$$
$$\text{or} \quad (x-2)^2+(y+3)^2=25$$

    b. Determine the equation of the circle in general form.

       The general form is $x^2+y^2+Dx+Ey+F=0$.

$$x^2-4x+4+y^2+6y+9=25$$
$$x^2+y^2-4x+6y-12=0 \quad \text{(general form)}$$

③ 2. Determine the centre coordinates and the radius of a circle which has the equation $x^2+y^2+6x-4y-3=0$.

$$\left(x^2+6x+\underline{}\right)+\left(y^2-4y+\underline{}\right)=3$$
$$\left(x^2+6x+9\right)+\left(y^2-4y+4\right)=3+9+4$$
$$(x+3)^2+(y-2)^2=16$$
$$\left[x-(-3)\right]^2+(y-2)^2=4^2$$
$$(x-h)^2+(y-k)^2=r^2$$
$$h=-3, k=2, r=4$$

      The centre of the circle is at $(h,\ k)=(-3,2)$.

           The radius $= 4$.

Topic 1

_____ marks

Topic 2 : Circle Equation

(2)   1.   Determine the standard form of the equation of a circle with the centre at $(-2, 4)$ and which passes through the point $(1, 3)$.

$$(h, k) = (-2, 4) \text{ and } (x, y) = (1, 3)$$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$[1 - (-2)]^2 + (3 - 4)^2 = r^2$$

$$(3)^2 + (-1)^2 = r^2$$

$$9 + 1 = r^2$$

$$r = \sqrt{10}$$

$$[x - (-2)]^2 + (y - 4)^2 = 10$$

The standard form is $(x + 2)^2 + (y - 4)^2 = 10$

or $[x - (-2)]^2 + (y - 4)^2 = (\sqrt{10})^2.$

③ 2. The endpoints of the diameter of a circle are located at $(-6, 3)$ and $(4, -7)$.
Determine the equation of this circle.
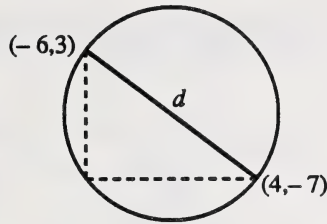
The centre of the circle is the midpoint of the diameter.

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left[\frac{(-6+4)}{2}, \frac{3+(-7)}{2}\right]$$

$$\text{midpoint} = (-1, -2) = (h, k) \qquad \text{(centre of circle)}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

Method 1	Method 2
$r = \frac{1}{2}d$	Use one of the two end points and the midpoint to find r.
$d = \sqrt{\Delta x^2 + \Delta y^2}$	
$d = \sqrt{[4-(-6)]^2 + (-7-3)^2}$	$(x-h)^2 + (y-k)^2 = r^2$
$d = \sqrt{10^2 + (-10)^2}$	$[-6-(-1)]^2 + [3-(-2)]^2 = r^2$
$d = \sqrt{200}$	$(-5)^2 + (5)^2 = r^2$
$d = 10\sqrt{2}$	$50 = r^2$
$r = \frac{1}{2}d = 5\sqrt{2}$	$(x-h)^2 + (y-k)^2 = r^2$
$(x-h)^2 + (y-k)^2 = r^2$	$[x-(-1)]^2 + [y-(-2)]^2 = 50$
$[x-(-1)]^2 + [y-(-2)]^2 = (5\sqrt{2})^2$	$(x+1)^2 + (y+2)^2 = 50$
$(x+1)^2 + (y+2)^2 = 25(2)$	
$(x+1)^2 + (y+2)^2 = 50$	

Topic 2

_____ marks

Topic 3 : Circles and Chords

(6) 1. Points A $(4, -4)$ and B$(2, -2)$ are on a circle, and the centre of the circle is on the line $3x - 4y - 17 = 0$. Determine the standard form of the equation of this circle.
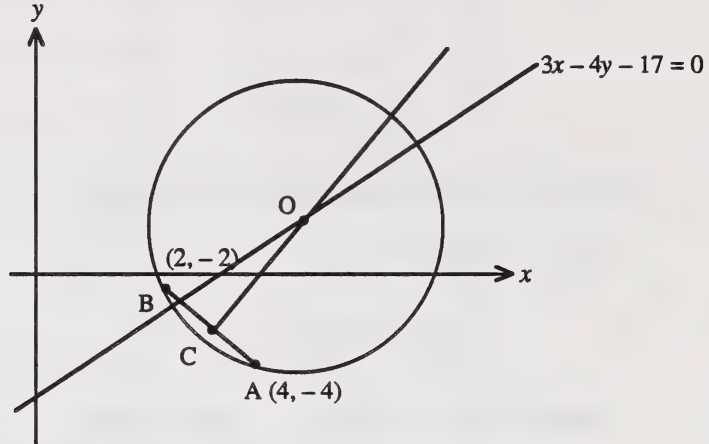
The equation of $\overline{CO}$ must be determined so that this equation and the given line equation $3x - 4y - 17 = 0$ can be solved to give the centre point of the circle. To obtain the equation of $\overline{CO}$, the slope of $\overline{CO}$ and the coordinates of C must be known. $\overline{CO} \perp \overline{AB}$. Thus, its slope is the negative reciprocal of the slope of $\overline{AB}$. C is the midpoint of $\overline{AB}$.

slope of $\overline{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4 - (-2)}{4 - 2} = \dfrac{-4 + 2}{2} = -1$

$\therefore$ slope of $\overline{CO} = 1$ (negative reciprocal of -1)

Coordinates of C $= \left(\dfrac{2+4}{2}, \dfrac{-2 + (-4)}{2} \right) = (3, -3)$

To obtain the equation for $\overline{CO}$, use the coordinates $(3, -3)$ and $m = 1$ and substitute into $y - y_1 = m(x - x_1)$.

$y - (-3) = 1(x - 3)$

$y + 3 = x - 3$

$x - y = 6$ (1)

By solving $x - y = 6$ and $3x - 4y = 17$, the coordinates of the centre point can be obtained.

$$x - y = 6 \quad \text{①}$$
$$3x - 4y = 17 \quad \text{②}$$
$$3 \times \text{①} \qquad 3x - 3y = 18 \quad \text{③}$$
$$\text{②} - \text{③} \qquad -1y = -1$$
$$y = 1$$
$$x - 1 = 6$$
$$x = 7$$

$\therefore$ centre of circle is $(7, 1) = (h, k)$.

Substitute point B$(2, -2)$ and (h, k) values $(7, 1)$ into $(x - h)^2 + (y - k)^2 = r^2$.

$$(2 - 7)^2 + (-2 - 1)^2 = r^2$$
$$(-5)^2 + (-3)^2 = r^2$$
$$25 + 9 = r^2$$
$$r^2 = 34$$
$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 7)^2 + (y - 1)^2 = 34$$

The equation of the circle is $(x - 7)^2 + (y - 1)^2 = 34$.

④

2. Points A $(4, -1)$, B $(2, -1)$, and C $(4, -3)$ are three points on a circle. Determine the equation of the circle.
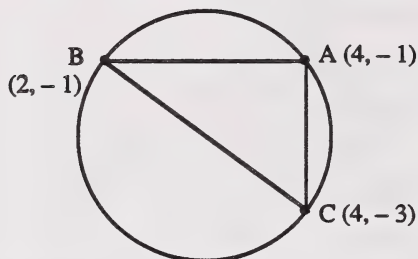
$\triangle$BAC is a right triangle. Coordinates point this out. For example, line $y = -1$ is perpendicular to line $x = 4$. This means the right $\triangle$ is inscribed in the semicircle so the midpoint of the hypotenuse is the centre of the circle.

$$\text{midpoint of } \overline{BC} = \left(\frac{2+4}{2}, \frac{-1+(-3)}{2} \right)$$

$$= (3, -2) = (h, k)$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-3)^2 + [y-(-2)]^2 = r^2$$

$$(x-3)^2 + (y+2)^2 = r^2$$

$(4, -1)$ is on this circle.

Use $(x-h)^2 + (y-k)^2 = r^2$.

$$(4-3)^2 + [-1-(-2)]^2 = r^2$$

$$1^2 + 1^2 = r^2$$

$$2 = r^2$$

The equation of the circle is $(x-3)^2 + (y+2)^2 = 2$.

Here is another method. Each pair of coordinates satisfies the equation.

A $(4,-1)$: $(4-h)^2+(-1-k)^2=r^2$

B $(2,-1)$: $(2-h)^2+(-1-k)^2=r^2$

C $(4,-3)$: $(4-h)^2+(-3-k)^2=r^2$

since $r^2=r^2$

① $(4-h)^2+(-1-k)^2=(2-h)^2+(-1-k)^2$

② $(4-h)^2+(-1-k)^2=(4-h)^2+(-3-k)^2=r^2$

From ① $\quad 16-8h+h^2+1+2k+k^2=4-4h+h^2+1+2k+k^2$

$$12=4h$$
$$3=h$$

From ② $\quad 16-8h+h^2+1+2k+k^2=16-8h+h^2+9+6k+k^2$

$$-8=4k$$
$$k=-2$$

Centre is $(h,k)=(3,-2)$.

Substitute $(3,-2)$ into any of the 3 original equations.

$$(4-3)+\left(\left[-1-(-2)\right]^2\right)=r^2$$
$$1^2+1^2=r^2$$
$$2=r^2$$

$$(x-3)^2+(y+2)^2=2$$

The two methods agree.

Topic 3

_____ marks

Topic 4 : Circles, Tangents, and Problem Solving

⑤

1. Determine the equation of a circle which has the centre (3, 4) and is tangent to the line
    $x+2y=21$.

    Change $x + 2y = 21$ to the slope y-intercept form.

    $$2y = -x + 21$$
    $$y = -\frac{1}{2}x + 10.5$$

    The slope is $-\frac{1}{2}$ and the y-intercept is (0, 10.5).
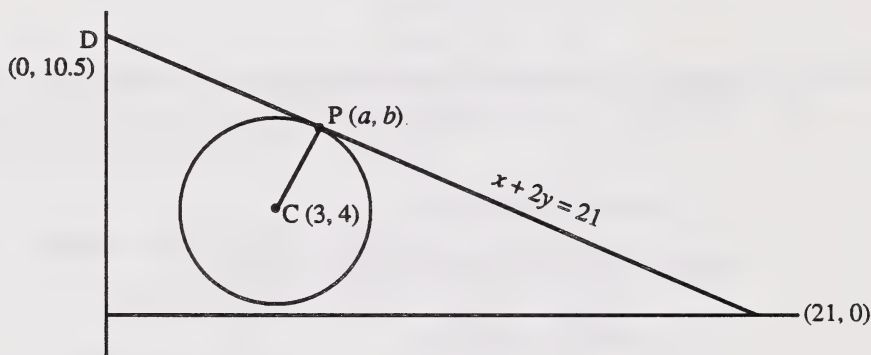
    Since the slope of the tangent line is $-\frac{1}{2}$, the slope of $\overline{PC} = 2$ (negative reciprocal). Now you
    have to obtain 2 equations with 2 unknowns to solve for (a, b).

    $$\text{Slope of } \overline{PC} = 2 = \frac{\Delta y}{\Delta x} = \frac{b-4}{a-3}$$
    $$2(a-3) = b-4$$
    $$2a-6 = b-4$$
    $$2a-b = 2 \quad \text{①}$$

    (a, b) also satisfies $x + 2y = 21$.

    $$a + 2b = 21 \quad \text{②}$$
    $$2 \times \text{①} \quad 4a - 2b = 4 \quad \text{③}$$
    $$\text{②} + \text{③} \quad 5a = 25$$
    $$a = 5$$

From ① $\quad\quad 2(5)-b=2$

$$b=8$$

$$(h,k)=(3,4)$$

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-3)^2+(y-4)^2=r^2$$

But $(a,b)=(5,8)$ and (a,b) must satisfy the equation of the circle.

$$(5-3)^2+(8-4)^2=r^2$$

$$2^2+4^2=r^2$$

$$20=r^2$$

$$r=2\sqrt{5}$$

$$(x-3)^2+(y-4)^2=\left(2\sqrt{5}\right)^2$$

or $\quad\quad (x-3)^2+(y-4)^2=20$

⑤

2. The line $3x + y - 110 = 0$ is tangent to a circle at the point P(35, 5), and the centre of the circle lies on the line $4x - y - 36 = 0$. Determine the equation of the circle and state the equation in the form $(x-h)^2 + (y-k)^2 = r^2$.

In order to find the centre location, two different equations that pass through the centre are required. One equation is given and the other must be determined from the radius line drawn from the tangent point P.
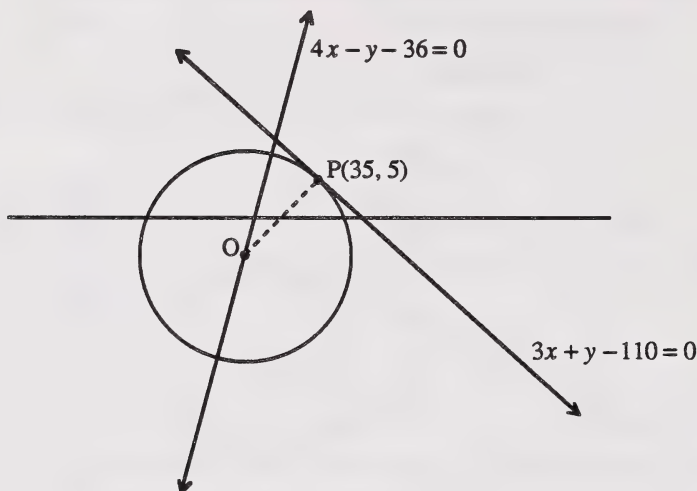
Since $3x + y - 110 = 0$ is tangent to the circle at P(35,5), the radius $\overline{PO}$ is perpendicular to $3x + y - 110 = 0$.

$$y = -3x + 110$$

Thus, the slope of the tangent line is -3. Thus, the slope of the radius $\overline{PO}$ is $\frac{1}{3}$ (negative reciprocal). The slope of $\overline{PO}$ is $\frac{1}{3}$ and it passes through (35, 5).
Thus, the equation of $\overline{PO}$ is

$$y - y_1 = m(x - x_1).$$

$$y - 5 = \frac{1}{3}(x - 35)$$

$$y - 5 = \frac{1}{3}x - \frac{35}{3}$$

$$3y - 15 = x - 35$$

$$x - 3y - 20 = 0$$

There are now two different equations which pass through the centre.

$$4x - y = 36 \quad ①$$
$$x - 3y = 20 \quad ②$$
$$4 \times ② \quad 4x - 12y = 80 \quad ③$$
$$① - ③ \quad 11y = -44$$
$$y = -4$$
$$4x - (-4) = 36$$
$$x = 8$$

The centre of the circle is $(8, -4)$ that is, (h, k).
The radius of the circle is from $(8, -4)$ to $(35, 5)$.

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$
$$r = \sqrt{(35-8)^2 + [5-(-4)]^2}$$
$$r = \sqrt{27^2 + 9^2} = \sqrt{729 + 81} = \sqrt{810}$$
$$r^2 = 810$$

$$(x-h)^2 + (y-k)^2 = r^2$$

The required equation is $(x-8)^2 + [y-(-4)]^2 = 810$
$$\text{or } (x-8)^2 + (y+4)^2 = 810.$$

Topic 4

_____ marks

Topic 5 : The Parabola

② 1. Determine the equation of a parabola with vertex at the origin and focus (3, 0).
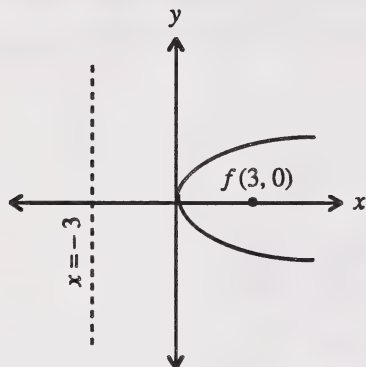
$y^2 = 4px,$ where $p = 3.$

$y^2 = 4(3)x$

$y^2 = 12x$

p is 3 since the distance from the vertex to the focus is 3 and the graph opens to the right.

The directrix is $x = -3.$

③ 2. Determine the equation of a parabola with vertex at the origin and the equation of the directrix is $y = 3$. State the coordinates of the focus.
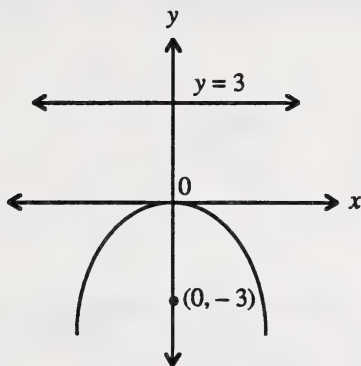
If the directrix is at $y = 3$, then the focus must be at $(0, -3)$. The graph opens downwards.

$$x^2 = 4py$$
$$x^2 = 4(-3)y$$
$$x^2 = -12y$$

p is the distance from the vertex to the focus and p is negative since the graph opens downwards.

The equation of the parabola is $x^2 = -12y$.

⑤   3. Determine the equation of a parabola with a horizontal axis of symmetry, a vertex (0, 0), and passing through the point (– 2, 4). Determine the length of the latus rectum and give the coordinates of the endpoints of the latus rectum.
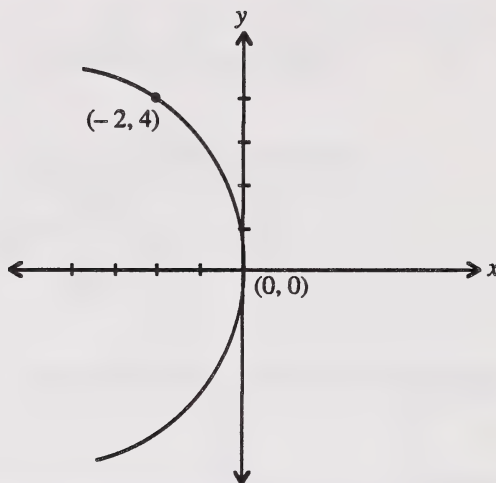
Since the vertex is at the origin, and the axis of symmetry lies along the x-axis, the standard form of the equation is $y^2 = 4px$.

Since (– 2, 4) is a point on the parabola, the equation is satisfied when $x = -2$ and $y = 4$. Substitute these values for x and y into the equation.

$$y^2 = 4px$$
$$(4)^2 = 4p(-2)$$
$$16 = -8p$$
$$p = -2$$

The required equation is $y^2 = 4(-2)x$

$$\text{or } y^2 = -8x.$$

The length of the latus rectum is $4|p| = 4|2| = 8$.
The latus rectum passes through the focus at (– 2, 0).
The endpoints of the latus rectum are (– 2, 4) and (– 2, – 4).

Topic 5

_____ marks

Topic 6 : The Parabola - Vertex Not at the Origin

②   1.   Determine the equation of the parabola when the focus is $(-3, -1)$ and the directrix is $y = 3$.
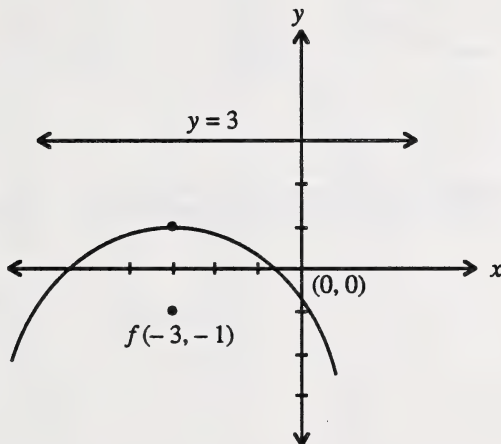
The distance between the directrix and the focus is 4 units. The vertex is half the distance between the directrix and the focus. Thus, the vertex is at $(-3, 1)$.

$\therefore (h, k) = (-3, 1)$.

$p = -2$ since the graph opens downward and the distance between the vertex and the focus is 2 units.

$$(x - h)^2 = 4p(y - k)$$
$$[x - (-3)]^2 = 4(-2)(y - 1)$$
$$(x + 3)^2 = -8(y - 1)$$

② 2. Determine the equation of the parabola which has a vertex at (– 3, 5) and the directrix is $x = 5$.
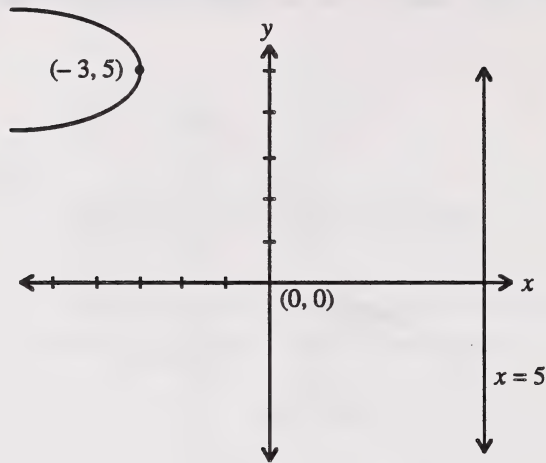
The vertex is at (– 3, 5). Thus, $(h, k) = (– 3, 5)$. The distance between the directrix and the vertex is 8 units. The focus is the same distance from the vertex as the vertex is from the directrix. Thus, $|p| = 8$. But p is negative since the parabola opens to the left. Thus, the coordinates of the focus are (– 11, 5).

This is a horizontal parabola which opens to the left.

$$(y-k)^2 = 4p(x-h)$$
$$(h, k) = (-3, 5)$$
$$(y-5)^2 = 4(-8)[x-(-3)]$$
$$(y-5)^2 = -32(x+3)$$

3. Determine the equation of a parabola which has a vertex at $(-2, 3)$ and which passes through the point $(-5, -3)$. This parabola has a vertical axis of symmetry.
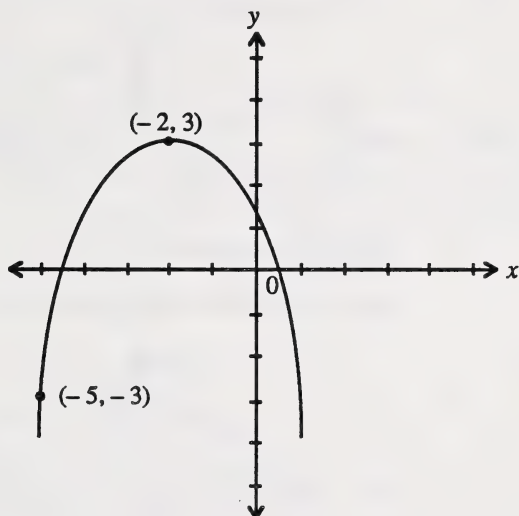
The vertex is at $(-2, 3)$. Thus, $(h, k) = (-2, 3)$.

$$(x-h)^2 = 4p(y-k)$$

$$[x-(-2)]^2 = 4p(y-3)$$

$$(x+2)^2 = 4p(y-3)$$

Now substitute $(-5, -3)$ into $(x+2)^2 = 4p(y-3)$.

$$(-5+2)^2 = 4p(-3-3)$$

$$(-3)^2 = 4p(-6)$$

$$9 = -24p$$

$$p = -\frac{9}{24}$$

The required equation is

$$(x+2)^2 = 4\left(-\tfrac{9}{24}\right)(y-3) \text{ or}$$

$$(x+2)^2 = -\tfrac{3}{2}(y-3).$$

The graph opens downward.

④ 4. A parabola has a horizontal axis of symmetry, a vertex at $(3, 4)$, and passes through the point $(-5, 1)$. Determine the equation of the parabola.
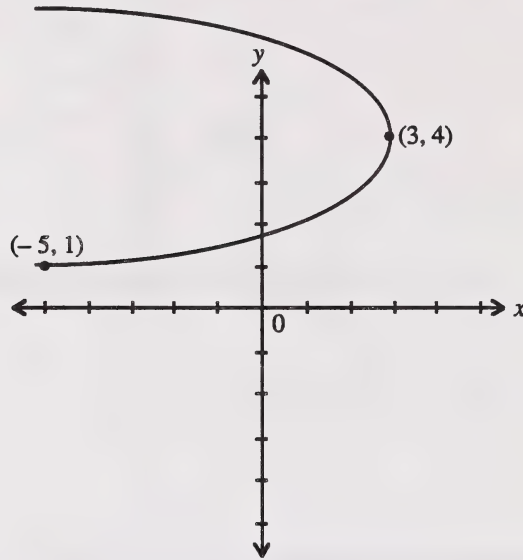
The horizontal axis of symmetry means it is a horizontal parabola of the form $(y-k)^2 = 4p(x-h)$ where $(h,k)=(3,4)$.

$$(y-4)^2 = 4p(x-3)$$

$(-5, 1)$ is on this parabola.

Thus, $(1-4)^2 = 4p(-5-3)$.

$$(-3)^2 = 4p(-8)$$
$$9 = -32p$$
$$p = -\frac{9}{32}$$

The equation of the directrix is $x = 3\frac{9}{32}$.

The required equation is $(y-4)^2 = 4\left(-\frac{9}{32}\right)(x-3)$

$$\text{or} \quad (y-4)^2 = -\frac{9}{8}(x-3).$$

The graph opens to the left.

③   5.   A tunnel is built in the shape of a parabolic arch as shown in the diagram below. The arch is 80 m wide at the base and the high point of the arch is 20 m above the baseline. Determine the equation of the parabola. Consider the high point of the arch as being the vertex with the coordinates (0, 0).

20 m

80 m

V (0, 0)
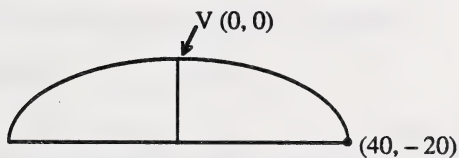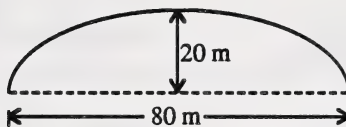
(40, – 20)

$$x^2 = 4py$$

Substitute $(40, -20)$.

$$40^2 = 4p(-20)$$
$$1600 = -80p$$
$$p = -20$$

The required equation is $x^2 = 4(-20)y$

or $x^2 = -80y$.

Topic 6

_____ marks

Topic 7 : The Ellipse - Centre at the Origin

(3) 1. The equation given below is for an ellipse centred at the origin with foci on the x-axis. Complete the table.

Equation in Standard Form	a	b	Coordinates of Vertices	Coordinates for Endpoints of Minor Axis	Length of Major Axis	Length of Minor Axis
$\dfrac{x^2}{49}+\dfrac{y^2}{36}=1$	7	6	$(7,0)$ $(-7,0)$	$(0,6)$ $(0,-6)$	$2a=14$	$2b=12$

(3) 2. The equation given below is for an ellipse centred at the origin with foci on the y-axis. Complete the table.

Equation in Standard Form	a	b	Coordinates of Vertices	Coordinates for Endpoints of Minor Axis	Length of Major Axis	Length of Minor Axis
$\dfrac{x^2}{25}+\dfrac{y^2}{81}=1$	9	5	$(0,9)$ $(0,-9)$	$(5,0)$ $(-5,0)$	$2a=18$	$2b=10$

③ 3. An ellipse centred at the origin is defined by the equation $9x^2 + 25y^2 = 225$.

Complete the following statements.

This equation in standard form is $\dfrac{x^2}{(5)^2} + \dfrac{y^2}{(3)^2} = 1$.

Since the denominator of the ___*x*___ - term is larger, this equation is of the

standard form $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

Now complete the blanks for the following questions.

a. For this ellipse, $a =$ __5__ and $b =$ __3__.

b. The major axis lies along the __*x*__-axis and is __10__ units long. The coordinates of the endpoints of the major axis are __(5, 0)__ and __(– 5, 0)__.

c. The minor axis lies along the __*y*__-axis and is __6__ units long. The coordinates of the endpoints of the minor axis are __(0, 3)__ and __(0, – 3)__.

d. The coordinates of the foci are __(4, 0)__ and __(– 4, 0)__.

$$c = \sqrt{a^2 - b^2}$$
$$c = \sqrt{25 - 9} = \sqrt{16} = 4$$

e. The length of the latus rectum is __3.6__ units.

The length $= \dfrac{2b^2}{a} = \dfrac{2(3)^2}{5} = \dfrac{18}{5} = 3.6$.

③ 4. The length of the major axis is 18 for an ellipse centred at the origin. The foci for this ellipse are located at $(3, 0)$ and $(-3, 0)$. Determine the standard form of the defining equation for this ellipse.

Since the foci are on the x-axis, the equation is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$$2a = 18 \qquad \text{(The length of the major axis is } 2a.)$$
$$a = 9$$
$$c^2 = a^2 - b^2$$
$$3^2 = 9^2 - b^2$$
$$b^2 = 9^2 - 3^2$$
$$b^2 = 81 - 9$$
$$b^2 = 72$$

Now substitute the values for a^2 and b^2 in the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

The equation of the ellipse is $\frac{x^2}{81} + \frac{y^2}{72} = 1$.

③   5. The equation $4y^2 = 36 - 9x^2$ defines an ellipse centred at the origin.
       Determine the coordinates of the foci and the coordinates of the vertices.

$$4y^2 = 36 - 9x^2$$

$$9x^2 + 4y^2 = 36$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

Since the denominator of the y^2-term is greater than the denominator of the x^2-term, the major axis is on the y-axis.

$$a = \sqrt{9} = 3, b = \sqrt{4} = 2$$

The coordinates of the vertices are $(0, 3)$ and $(0, -3)$.

$$c = \sqrt{a^2 - b^2}$$
$$= \sqrt{9 - 4} = \sqrt{5}$$

Thus, the coordinates of the foci are $\left(0, \sqrt{5}\right)$ and $\left(0, -\sqrt{5}\right)$.

④

6. Determine the equation of an ellipse in standard form where the vertices are at $(0, \pm 8)$ and the ellipse passes through the point $(3, 2)$.

The vertices are on the y-axis; thus, the equation is of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

Substitute 8 for a.

$$\frac{x^2}{b^2} + \frac{y^2}{8^2} = 1$$

$(3, 2)$ is on the ellipse. Substitute $x = 3$ and $y = 2$ in the equation.

$$\frac{3^2}{b^2} + \frac{2^2}{64} = 1$$

$$\frac{9}{b^2} = 1 - \frac{4}{64} = 1 - \frac{1}{16}$$

$$\frac{9}{b^2} = \frac{15}{16}$$

$$b^2 = \frac{9 \times 16}{15} = \frac{144}{15} = \frac{48}{5}$$

∴ The required equation is $\dfrac{x^2}{\frac{48}{5}} + \dfrac{y^2}{64} = 1$.

Topic 7

_____ marks

Topic 8 : The Hyperbola - Centre at the Origin

⑤   1. A hyperbola centred at the origin has the equation $16x^2 - 4y^2 = 64$.
Complete the questions by filling in the blank spaces.

   a.   This equation in standard form is $\dfrac{x^2}{(2)^2} - \dfrac{y^2}{(4)^2} = 1$.

   This equation is of the general form $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.

   b.   For this hyperbola, $a = \underline{\ 2\ }$ and $b = \underline{\ 4\ }$.

   c.   The transverse axis lies along the $\underline{\ x\ }$-axis. The transverse axis is $\underline{\ 4\ }$
   units long. The coordinates of the vertices are $\underline{\ (2, 0)\ }$ and $\underline{(-2, 0)}$.
   The vertices are also the $\underline{\ x\ }$-intercepts.

   d.   The conjugate axis lies along the $\underline{\ y\ }$-axis. The conjugate axis is $\underline{\ 8\ }$
   units long. The coordinates of the endpoints of the conjugate axis are
   $\underline{\ (0, 4)\ }$ and $\underline{\ (0, -4)\ }$.

   e.   The foci lie on the $\underline{\ x\ }$-axis. The coordinates of the foci are
   $\underline{\ (2\sqrt{5}, 0)\ }$ and $\underline{\ (-2\sqrt{5}, 0)\ }$.

   $c = \sqrt{a^2 + b^2}$

   $c = \sqrt{2^2 + 4^2}$

   $ = \sqrt{20}$

   $ = 2\sqrt{5}$

   f.   The length of the latus rectum is $\underline{\ 16\ }$.

   $\dfrac{2b^2}{a} = \dfrac{2(4)^2}{2} = 16$

   g.   The equations of the asymptotes are $y = \pm 2x$.

   $y = \pm\dfrac{b}{a}x = \pm\dfrac{4}{2}(x)$

   h.   The hyperbola opens $\underline{\ \text{right}\ }$ and $\underline{\ \text{left}\ }$.

③ 2. Determine the coordinates of the foci of a hyperbola centred at the origin where the foci are located on the *y*-axis. The equation of the hyperbola is $\frac{x^2}{25} - \frac{y^2}{16} = -1$.

The standard form for this equation is $\frac{x^2}{b^2} - \frac{y^2}{a^2} = -1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

$$b^2 = 25 \qquad a^2 = 16$$
$$b = 5 \qquad a = 4$$

$$c^2 = a^2 + b^2$$
$$c^2 = 16 + 25$$
$$c^2 = 41$$
$$c = \pm\sqrt{41}$$

The foci are at $\left(0,\ \sqrt{41}\right)$ and $\left(0, -\sqrt{41}\right)$.

④ 3. Determine the equation of a hyperbola in standard form if the foci are at $(0,\pm6)$ and the length of the transverse axis is 8 units.

Since the foci are on the y-axis, the defining equation is of the form $\frac{x^2}{b^2}-\frac{y^2}{a^2}=-1$ or $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$.
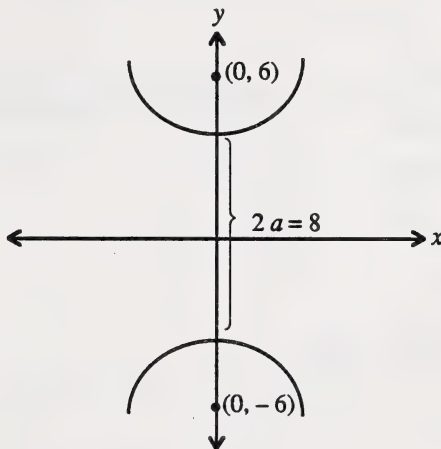
$c=6$ and $2a=8$. Thus, $a=4$.

$c^2=a^2+b^2$

$6^2=4^2+b^2$

$b^2=36-16=20$

The equation is $\frac{x^2}{20}-\frac{y^2}{16}=-1$ or $\frac{y^2}{16}-\frac{x^2}{20}=1$.

④ 4. Determine the equation of a hyperbola in standard form if the foci are at $(\pm 7, 0)$ and the x-intercepts are at (± 3).

Since the foci are on the x-axis, the defining equation is of the form $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.

$c = 7, a = 3$

$c^2 = a^2 + b^2$

$7^2 = 3^2 + b^2$

$b^2 = 49 - 9$

$b^2 = 40$

The equation is $\dfrac{x^2}{9} - \dfrac{y^2}{40} = 1$.

④ 5. Determine the equation of a hyperbola in standard form if the vertices are at $(\pm 6, 0)$ and the hyperbola passes through the point (8, 3).

Since the vertices are on the x-axis, the defining equation is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

When $a = 6$, $\frac{x^2}{6^2} - \frac{y^2}{b^2} = 1$.

The point (8, 3) is on the hyperbola. Thus, $\frac{8^2}{6^2} - \frac{3^2}{b^2} = 1$.

$$\frac{64}{36} - \frac{9}{b^2} = 1$$

$$\frac{64}{36} - \frac{36}{36} = \frac{9}{b^2}$$

$$\frac{28}{36} = \frac{9}{b^2}$$

$$b^2 = \frac{9 \times \overset{9}{\cancel{36}}}{\underset{7}{\cancel{28}}} = \frac{81}{7}$$

The equation is $\dfrac{x^2}{36} - \dfrac{y^2}{\frac{81}{7}} = 1$.

Topic 8

_____ marks

Topic 9 : Quadratic Relations and Conic Sections

②

1. For each of the following equations tell whether the graph is a circle, ellipse, parabola, or hyperbola.

   a. $2x^2 + 6y^2 - 8 = 0$

   The general form equation of an ellipse centred at the origin is $Ax^2 + Cy^2 + F = 0$ where $A \neq C$, $A > 0$, and $C > 0$.

   It is an ellipse centred at the origin, $A \neq C$, $A > 0$, $C > 0$, and $D = E = 0$.

   b. $3x^2 - 7y^2 + 8 = 0$

   The general form equation of a hyperbola centred at the origin is $Ax^2 + Cy^2 + F = 0$, where $A > 0$ and $C < 0$ or $A < 0$ and $C > 0$.

   It is a hyperbola centred at the origin. A and C are of opposite signs.

③ 2. Determine the equation defined by the locus of all points equidistant from a fixed point F (2, 0) and the line $x = 8$.
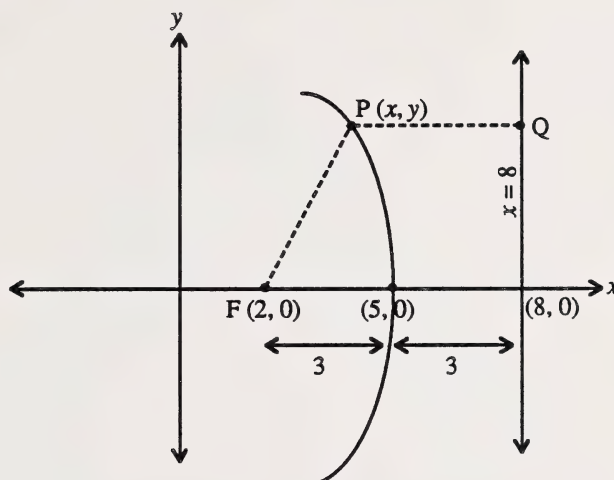
The information given represents a parabola. This is a horizontal parabola opening to the left since the directrix is to the right of the focus. Any point on the curve satisfies the locus definition PF = PQ.

$$PF = \sqrt{(x-2)^2+y^2} \text{ and } PQ = 8-x$$

Since PF = PQ, $(x-2)^2+y^2 = (8-x)^2$ when both sides are squared.

$$x^2-4x+4+y^2 = 64+x^2-16x$$

$$y^2+12x-60 = 0$$

The equation $y^2+12x-60 = 0$ should satisfy point (5, 0) which is the vertex of the parabola. The vertex is located halfway between the focus and directrix.

$$y^2 = -12x+60$$

When $x = 5$,

$$y^2 = -12(5)+60.$$

$$y = 0$$

The equation is $y^2+12x-60 = 0.$

Topic 9

_____ marks
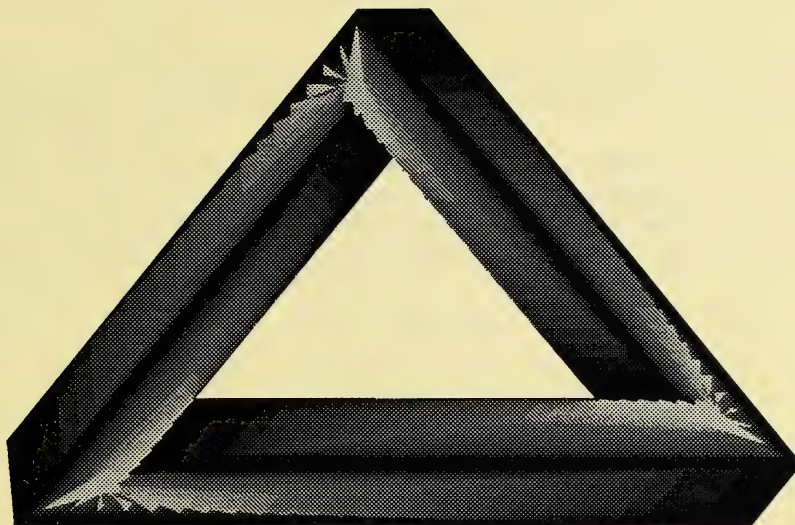
MATHEMATICS 30

LEARNING FACILITATOR'S MANUAL

UNIT 6

Distance Learning

Alberta EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed
assignments and the final test; therefore, it should be kept secure by the teacher. Students
should not have access to these assignments or the final tests until they are assigned in a
supervised situation. The answers should be stored securely by the teacher at all times.

Mathematics 30
Learning Facilitator's Manual
Unit 6
Statistics
Alberta Distance Learning Centre
ISBN No. 0-7741-0099-0

Topic 1: The Normal Distribution

At the back of this Assignment Booklet is a table of **Areas under the Standard Normal Curve.** You may need this to complete some of the following questions.

1. The following numbers are the test scores from a sample of fifty students. Develop a frequency distribution which consists of ten classes and use this distribution to calculate the mean, mode, and median.

58	63	68	77	90	53	43	55	47	60
57	76	65	70	81	28	49	73	88	67
36	75	72	87	96	40	46	51	57	61
42	76	83	66	57	39	48	54	62	86
52	60	85	72	45	59	43	68	47	51

①   a. Determine the range for these test scores.

$$\text{range} = 96 - 28$$
$$= 68$$

①   b. Calculate the class width, i.

Class width = range ÷ number of classes
$$i = \underline{68} \div 10$$
$$= 6.8$$
$$= 7 \ (\text{to the next whole number})$$

②   c. Calculate the class limits and the class boundaries for each class interval.

First class interval:
$$\text{lower limit} = \text{lowest score}$$
$$= 28$$
$$\text{upper limit} = \text{lower limit} + (i - 1)$$
$$= 28 + (7 - 1)$$
$$= 34$$

The first interval is (28 - 34).

Class boundaries are halfway between the upper limit of one class and the lower limit of the next class. The class boundaries for the first interval are 27.5 and 34.5. Calculate the class limits and boundaries for the remaining intervals. Enter these values in the frequency distribution table in part h of this problem.

(1) d. The class mark, x, or midpoint for each class interval is the average of the class limits.
The class mark for the first class interval is $x = \dfrac{28+34}{2} = \underline{31}$.

Compute the class marks for the remaining intervals and enter these values in the frequency distribution table.

(1) e. Tally the test scores on a separate piece of paper in order to determine the frequency for each class interval. Then, enter these frequencies in the frequency distribution table.

(1) f. Calculate the product $(f \cdot x)$ for each class interval in the frequency distribution table. This column is used in the calculation of the mean. Add the ten values which give the value of $\sum (f \cdot x)$ shown at the bottom of the $(f \cdot x)$ column.

(1) g. Complete the cumulative frequency column which is used in the calculation of the median.

h. The values for the first two class intervals have been given in the following frequency distribution table. The reason for giving these values is to provide you with the proper start for completing the table.

Class Interval	Class Limits	Class Boundaries	Midpoint or Class Mark (x)	Frequency (f)	Products ($f \cdot x$)	Cumulative Frequency
1	28 - 34	27.5 - 34.5	31	1	$1 \times 31 = 31$	1
2	35 - 41	34.5 - 41.5	38	3	$3 \times 38 = 114$	4
3	42 - 48	41.5 - 48.5	45	8	$8 \times 45 = 360$	12
4	49 - 55	48.5 - 55.5	52	7	$7 \times 52 = 364$	19
5	56 - 62	55.5 - 62.5	59	9	$9 \times 59 = 531$	28
6	63 - 69	62.5 - 69.5	66	6	$6 \times 66 = 396$	34
7	70 - 76	69.5 - 76.5	73	7	$7 \times 73 = 511$	41
8	77 - 83	76.5 - 83.5	80	3	$3 \times 80 = 240$	44
9	84 - 90	83.5 - 90.5	87	5	$5 \times 87 = 435$	49
10	91 - 97	90.5 - 97.5	94	1	$1 \times 94 = 94$	50
				$\sum f = n =$ 50	$\sum (f \cdot x) =$ 3076	

① i. Determine the mean ($\bar{x}$) to two decimal places.

$$\bar{x} = \frac{\sum (f \cdot x)}{n} = \frac{3076}{50}$$
$$= 61.52$$

①   j.   Determine the mode.

Examine the frequency column and note that the highest frequency is ___9___ and this frequency occurs in the interval _____56 - 62_____. The midpoint of this interval is ___59___ so the mode is ___59___.

①   k.   Determine the median to two decimal places.

The cumulative frequency of the median is half the total cumulative frequency. The cumulative frequency of the median is $\frac{1}{2}(50) = 25$. A cumulative frequency of 25 is located in the interval 56 - 62. The boundaries of this interval are 55.5 and 62.5.

Interval with Class Limits 56 - 62

	Boundaries	Cumulative Frequency
(lower boundary)	55.5	19
	↓	↓
	y	25
	↓	↓
(upper boundary)	62.5	28

$$\frac{y - 55.5}{62.5 - 55.5} = \frac{25 - 19}{28 - 19}$$

$$\frac{y - 55.5}{7} = \frac{6}{9}$$

$$y = \underline{60.17}$$

The median is ___60.17___.

(4)

2. To determine the standard deviation (σ) for the data in question 1, three new columns with headings $(x-\bar{x})$, $(x-\bar{x})^2$, and $f \cdot (x-\bar{x})^2$ are required. Complete these values in the following table, but round your mean value ($\bar{x}$) from part i to the nearest whole number before determining the values. Round your final answer to one decimal place.

Class Interval	x	f	$(x-\bar{x})$	$(x-\bar{x})^2$	Products $f \cdot (x-\bar{x})^2$
1	31	1	-31	961	961
2	38	3	-24	576	1728
3	45	8	-17	289	2312
4	52	7	-10	100	700
5	59	9	-3	9	81
6	66	6	4	16	96
7	73	7	11	121	847
8	80	3	18	324	972
9	87	5	25	625	3125
10	94	1	32	1024	1024

$$\sum f \cdot (x-\bar{x})^2 = 11\ 846$$

$$\sigma = \sqrt{\frac{\sum f \cdot (x-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{11\ 846}{50}}$$

$$= \sqrt{236.92}$$

$$= 15.4$$

② 3. Temperature readings for a certain day in August over a period of years have a mean reading of 22°C and a standard deviation of 1.5°C. Complete the following normal curve diagram and assume normal distribution. Then, use this diagram to answer the questions that follow.
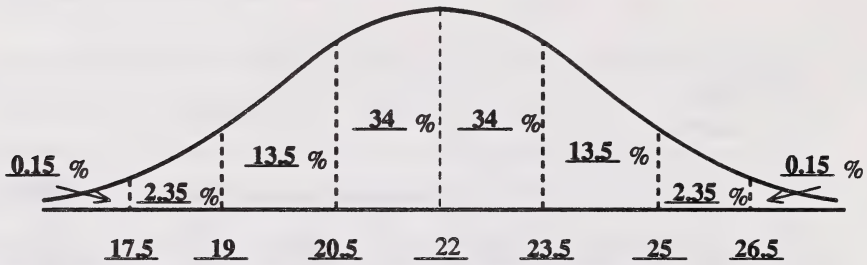
a. What percent of the readings will be between 17.5°C and 20.5°C?

$$2.35\% + 13.5\% = 15.85\%$$

b. What percent of the data is within two standard deviations of the mean?

$$2(34\% + 13.5\%) = 2(47.5\%)$$
$$= 95.0\%$$

c. What percent of the data is greater than 20.5°C?

$$100\% - (13.5\% + 2.35\% + 0.15\%) \text{ or } 34\% + 34\% + 13.5\% + 2.35\% + 0.15\% = 84\%$$

②   4. Cookies packaged in boxes have a mean mass of 180 g. The standard deviation is 2 g.

    a. When 8000 boxes are produced, how many boxes are within 2 g of the required mass?

$$0.68 \times 8000 = 5440$$

When 8000 boxes are produced, 5440 will be within 2 g of the required mass.

    b. How many boxes will not be accepted if the boxes must have a mass between 176 g and 184 g?

$$(2.5\% + 2.5\%) = 5\%; \text{ thus, } \frac{5}{100} \times 8000 = 400.$$

Four hundred boxes will not be accepted.

⑦

5. A cordless grass shear manufacturer determines the mean life of its shears to be 5.5 years, with a standard deviation of 1.5 years.

   a. If the manufacturer guarantees the shears for one year, what percentage of the shears will have to be replaced?

1.0   2.5   4   5.5   7   8.5

— % to be replaced is 0.15% since
   $50\% - (34\% + 13.5\% + 2.35\%) = 0.15\%$.
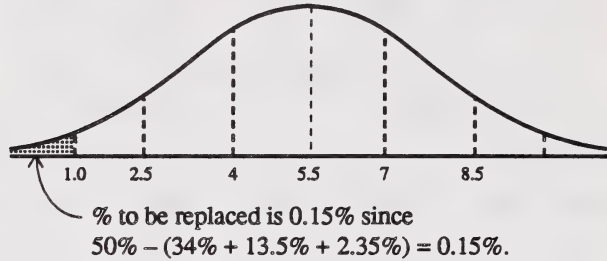
The percentage that will need to be replaced is 0.15%.

   b. If 4000 shears are sold to stores, how many shears will have to be replaced within one year?

Check by formula and z-score table.

$\dfrac{0.15}{100} \times 4000 = 6$

Six shears will have to be replaced.

$z = \dfrac{x - \mu}{\sigma}$

$z = \dfrac{1 - 5.5}{1.5}$

$z = \dfrac{-4.5}{1.5}$

$z = -3$

The area for $z = -3$ is 0.4987.

Required area $= 0.5 - 0.4987 = 0.0013 = 0.13\%$

The table is set for 34.13, 13.59, 2.15, and 0.13 not for 34, 13.5, 2.35, and 0.15.

By the z-score method the number of shears that have to be replaced is $\dfrac{0.13}{100} \times 4000 = 5$.

① 6. Some test scores are normally distributed with a mean of 90 and a standard deviation of 8. What percentage of the test scores fall between 74 and 98?
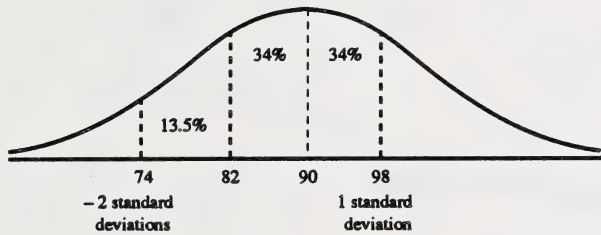
Method Using z-Scores

34% + 34% + 13.5% = 81.5%

$$z_1 = \frac{74 - 90}{8} \qquad z_2 = \frac{98 - 90}{8}$$

$$z_1 = \frac{-16}{8} \qquad z_2 = 1$$

$$z_1 = -2$$

Use the z-score table to obtain the area from $z = 1$ to $z = -2$.

$$Area = 0.4772 + 0.3413$$
$$= 0.8185$$
$$= 81.85\%$$

② 7. A student obtains a mark of 84 on a math exam. The mean mark for this math exam is 76 and the standard deviation is 12. This same student obtains a mark of 88 on a physics exam. For the physics exam, the mean mark is 79 and the standard deviation is 16. In which course does the student have a better relative standing?

z for math	z for physics

$$z = \frac{84 - 76}{12} \qquad\qquad z = \frac{88 - 79}{16}$$

$$z = 0.67 \qquad\qquad\qquad z = \frac{9}{16}$$

$$z = 0.56$$

The student has a better relative standing in mathematics.

①　8.　The mean grade for a test was 67 and the standard deviation was 9.　If a student had a z-score of 0.78, what was the student's test mark?

$$z = \frac{x - \mu}{\sigma}$$

$$0.78 = \frac{x - 67}{9}$$

$$x - 67 = 9(0.78)$$

$$x = 67 + 7.02$$

$$x = 74$$

The student's test mark was 74%.

④ 9. A battery company determines the mean life of its car batteries to be forty-eight months with a standard deviation of six months.

a. What is the probability that a battery will last longer than sixty months?

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{60 - 48}{6} = \frac{12}{6} = 2$$
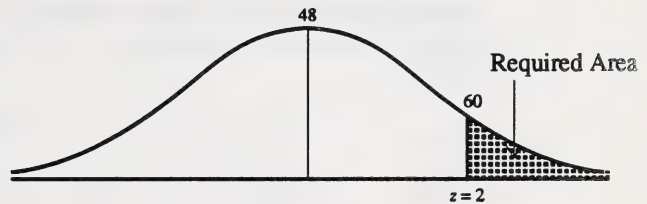
The area for $z = 2$ is 0.4772.
The required area is $0.5 - 0.4772 = 0.0228$.
The probability that a battery lasts longer than sixty months is 0.0228.

Approximate Area Method:

Required area $= 0.5 - (0.34 + 0.135)$

$= 0.025$

The probability is 0.025.

b. If the company guarantees the battery for forty months, what percentage of batteries will have to be replaced?
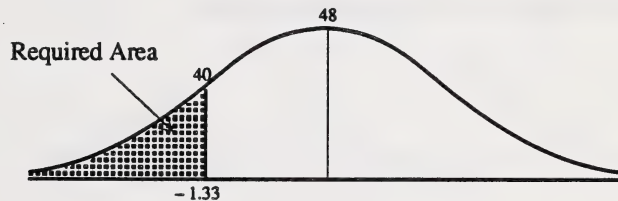
$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{40 - 48}{6} = \frac{-8}{6} = -1.3\dot{3}$$

The area for $z = -1.33$ is 0.4082.
The required area is $0.5 - 0.4082 = 0.0918$.
The percentage of batteries that will need to be replaced is 9.18%.

③ 10. The speeds of vehicles in a 60 km/h zone are normally distributed about a mean of 63 km/h with a standard deviation of 3 km/h. The police must allow 10% of the posted speed limit in case of error. If 120 cars pass through a radar speed trap in which the posted speed limit is 60 km/h, determine the number of cars stopped for speeding. Assume that all the cars travelling 10% or more above the speed limit will be ticketed.

Since $60 + 0.10\,(60) = 60 + 6 = 66$, cars going 66 km/h or faster will be stopped for speeding.

$$z = \frac{x - \mu}{\sigma}$$
$$= \frac{66 - 63}{3}$$
$$= 1$$
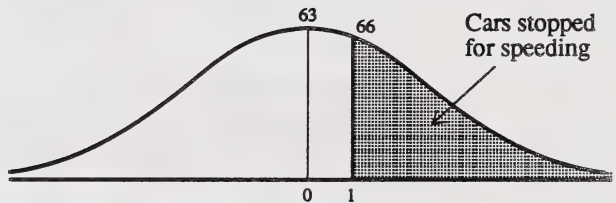
The area for $z = 1$ is 0.3413.

Required area $= 0.5000 - 0.3413$
$$= 0.1587$$

$0.1587 \times 120 = 19.044$

Thus, nineteen cars are stopped for speeding.

② 11. A clothes dryer manufacturer determines the mean life of a dryer to be eighty-six months with a standard deviation of thirty-six months. What guarantee, to the nearest month, should the manufacturer offer if it is only willing to repair 3% of the machines sold?

$$50\% - 3\% = 47\%$$

$$z\text{-score for } 0.47 = -1.88$$

$$z = \frac{x - \mu}{\sigma}$$

$$-1.88 = \frac{x - 86}{36}$$

$$-67.68 = x - 86$$

$$86 - 67.7 = x$$

$$18.3 = x$$

The guarantee should be for eighteen months.

Check

Let $x = 18$.

$$z = \frac{18 - 86}{36}$$

$$= \frac{-68}{36}$$

$$= -1.89$$

Area $= 0.4706$

Required area $= 0.5 - 0.47$

$$= 0.03$$

$$= 3\%$$

This agrees with the percentage of machines that the manufacturer is willing to repair.
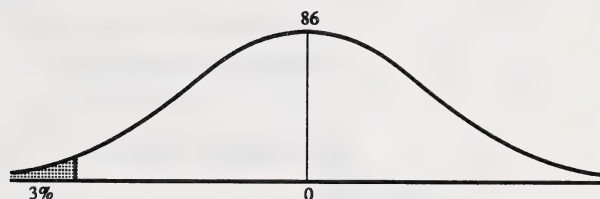
(2)

12. A manufacturer of electric lawn and gardening tools determines that the mean life of its best hedge trimmer is fifteen years, and the standard deviation is four years. What is the probability that a hedge trimmer must be returned within six years?
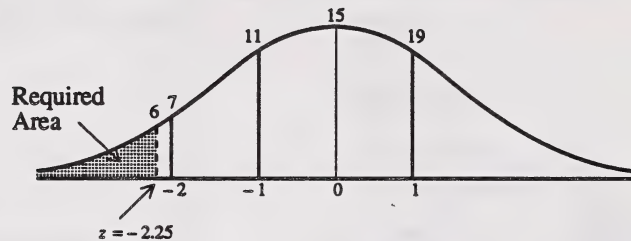
$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{6 - 15}{4}$$

$$z = \frac{-9}{4}$$

$$z = -2.25$$

The area for $z = -2.25$ is 0.4878.

Required area $= 0.5 - 0.4878$

$$= 0.0122$$

The probability is 0.0122.

(4)   13.   In a survey of 200 male students it is determined that their mean height is 170 cm and the standard deviation is 4 cm. Assume a normal distribution for the heights and answer the following questions.

a.   How many students have heights of 167 cm or less?

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{167 - 170}{4}$$

$$= -\frac{3}{4}$$

$$= -0.75$$
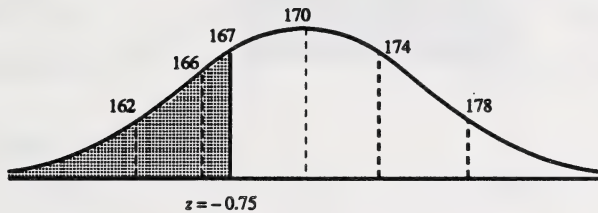
The area for $z = -0.75$ is 0.2734.
The required area is $0.5 - 0.2734 = 0.2266$.

Thus, $0.2266 \times 200 = 45.32$.
Thus, forty-five students have heights 167 cm or less.

b.   How many students have heights of 177 cm or more?

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{177 - 170}{4}$$

$$= \frac{7}{4}$$

$$= 1.75$$
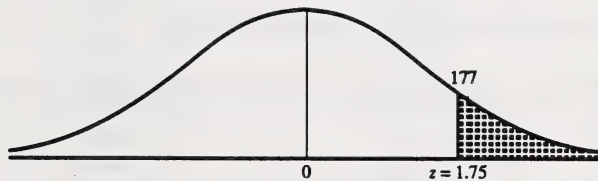
The area for $z = 1.75$ is 0.4599.
The required area is $0.5 - 0.4599 = 0.0401$.

Thus, $0.0401 \times 200 = 8.02$.
Thus, eight students have heights 177 cm or more.

c. How many students have heights between 165 cm and 178 cm?
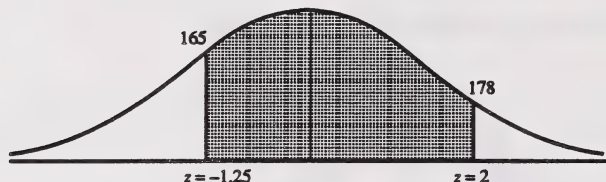
165 cm	178 cm
$z = \dfrac{165 - 170}{4}$	$z = \dfrac{178 - 170}{4}$
$= -\dfrac{5}{4}$	$z = 2$
The area for $z = -1.25$ is 0.3944.	The area for $z = 2$ is 0.4772.

The total area is $0.3944 + 0.4772 = 0.8716$.

$$0.8716 \times 200 = 174.32$$

Thus, 174 students have heights between 165 cm and 178 cm.

d. Sixteen percent of the students are below what height?

To find 16% below a certain height, use 34%. From the tables an area of 0.34 gives a z-score of -1.

$z = \dfrac{x - \mu}{\sigma}$

$-1 = \dfrac{x - 170}{4}$ (-0.995 rounded to two decimal places is -1.00.)

$-4 = x - 170$

$x = 166$ cm   Sixteen percent of the students are below 166 cm.

Topic 1

_____ marks

Topic 2: Bivariate Data

①

1. Circle the correct response.
   The science of collecting and classifying data in order to show their significance is called

   a. data processing

   (b.) statistics

   c. technology

   d. mathematics

   The correct response is b.

2. The following table of marks shows the relationship between the work habits and the average marks for students at a college.

Students	Rating of Work Habits	Average Mark
Marie	30	10
Sue	10	30
Joe	30	40
Gary	10	50
Gurmeet	50	30
Sandy	60	60
Lou	60	70
Jerry	40	80
Maria	80	80
Rhonda	80	50
Olga	100	70
Joyce	100	90
Chen	90	90
Roy	20	10

②   a.   Draw a scatterplot of the work habits of the students against their average marks. Place the work habits along the horizontal axis.
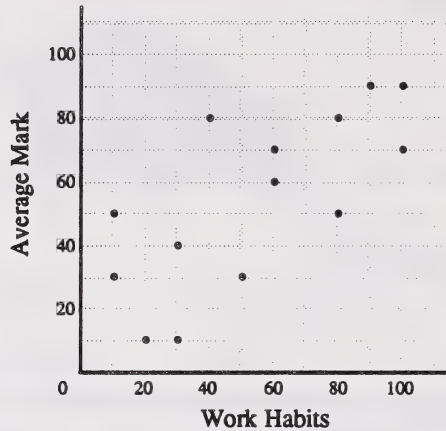
①   b.   Is there a positive correlation, negative correlation, or no correlation?

There is a strong positive correlation between work habits and the average mark. Students with good work habits generally have higher averages.

②   c.   Discuss the performances of Sandy, Maria, and Chen.

Sandy, Maria, and Chen all have the same ratio of work habit rating to average mark. They are consistent. However, they are not performing equally well. Sandy is much lower than Chen as far as achievement is concerned.

②   d.   How do the results of Gary and Jerry vary?

Gary's mark was average in the course, yet he performed very low on the work habit rating. This would seem to suggest that there is no relationship between his average score and his work habit rating.

In the case of Jerry, his average mark is fairly high while his work habit rating is low. This shows that you cannot always be sure about the relationship as predicted by the correlation coefficient.

③   3.   Find the line of best fit for the following scatterplot using the median fit method.
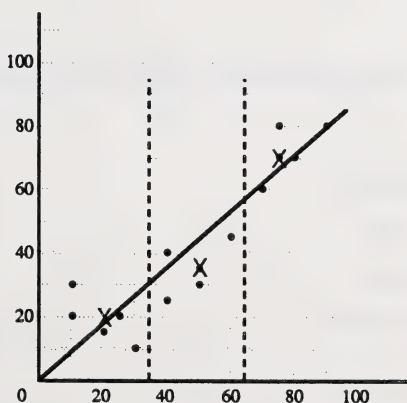
4. Determine the equation for the line of best fit AD on the following scatterplot.
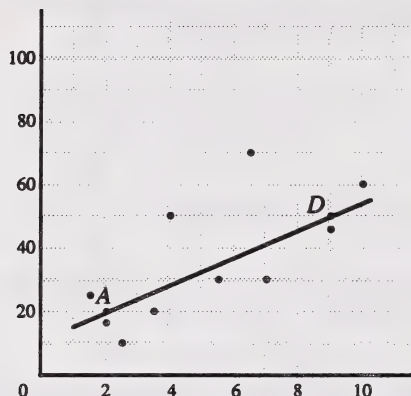
Use points $A(2, 20)$ and $D(9, 50)$ to find the slope of the line of best fit.

$$m = \frac{y_0 - y_1}{x_0 - x_1}$$
$$m = \frac{50 - 20}{9 - 2}$$
$$m = \frac{30}{7}$$

Substitute the slope and the coordinates of point D into the point slope form of the linear equation.

$$y - y_1 = m(x - x_1)$$
$$y - 50 = \frac{30}{7}(x - 9)$$
$$7y - 350 = 30x - 270$$
$$0 = 30x - 7y + 80$$

The equation of the line is $30x - 7y + 80 = 0$.

5. Circle the correct response.
   Which of the following is the range of the correlation coefficient?

   (a.) $-1 \leq r \leq 1$

   b. $0 \leq r \leq 1$

   c. $-1 < r \leq 1$

   d. $-1 \leq r \leq 0$

   The correct response is a. The correlation coefficient may be -1 for a strong negative correlation and $+1$ for a strong positive correlation.

6. Circle the correct response.
   Which of the following graphs is an example of a strong negative correlation?

a.

(b.)

c.

d.
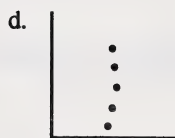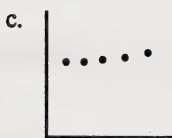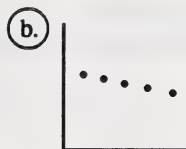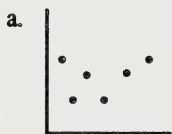
The correct response is b.

② 7. Draw the line of best fit for the following scatterplot.
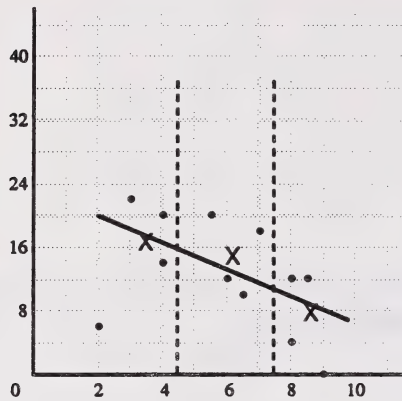
① 8. Circle the correct response.
Which of the following is the weakest correlation coefficient?

a. + 0.6

b. − 0.55

c. + 0.12

d. + 0.04

The correct response is d. There is very little correlation between the two sets of data when the correlation coefficient is 0.04.

① 9. Circle the correct response.
Which of the following indicates how well the line of best fit demonstrates the situation?

   a. standard deviation

   (b.) correlation coefficient

   c. confidence coefficient

   d. sample of the population

   The correct response is b.

Topic 2

_____ marks

Topic 3: Confidence

Circle the correct response for questions **1** to **3**.

①

1. The proportion of the number of yeses in a population is called the

    (a.) population percentage

    b.   sample proportion

    c.   confidence interval

    d.   sampling distribution

    The correct response is a. The population percentage is the percentage (or proportion) of objects in the population that are yeses.

①

2. Which type of sampling is not a type of convenience sampling?

    a.   self-selected

    b.   judgement

    (c.) clustered

    d.   biased

    The correct response is c. At no time does the surveyor decide who will be included in the sample; thus, clustered sampling is not a type of convenience sampling.

①   3.   A sample in which an expert selects the sample based on knowledge of the population is called

      a.   systematic sampling

      b.   self-selected sampling

      c.   clustered sampling

      d.   judgement sampling

      The correct response is d.

⑫   4.   Explain any four of the following.

      a.   correlation

         The correlation between two or more variables is a measure of the relationship between the variables. If two variables are correlated, they generally have a lot in common. Mathematics and science, for example, should have a strong correlation. On a grid, a positive correlation has its graph rising to the right. A negative correlation has its graph falling to the right.

         A measure of how well two variables are related is given by the correlation coefficient. The range of this coefficient is between -1 and $+1$ or $-1 \leq r \leq 1$.

         It should be noted that -1 represents a very strong negative correlation and $+1$ represents a very strong positive correlation.

b. bivariate

A set of data that contains two variables is said to be bivariate. For example, if you compare the wingspan of an aircraft with the cost of the aircraft, you would be dealing with two variables that are not closely related yet have a correlation. The correlation may be negative or positive, but the correlation does exist.

A study of this bivariate data would produce a scatterplot graph, and a line of best fit would indicate the correlation.

c. line of best fit

Sometimes the points on a scatterplot graph appear to be clustered about a line. The straight line that can be drawn joining the greatest possible number of points on the scatterplot is called the line of best fit.

The slant of the distribution can be determined by this line. The line of best fit is usually sketched so that an equal number of points lie on either side of the line. If the points are close to the line, you have strong correlation between the variables. A strong correlation may be either negative or positive.

d. measures of central tendency

The three main measures of central tendency are the mean, mode, and median. They are good indicators as to the distribution of the data. Each gives a different measure of how the data is grouped around the centre. They also summarize the distribution of a sample. If the mean, mode, and median are close to each other, the distribution is said to be normal.

e. 95% box and whisker plots

Surveys and polls are often designed to collect yes or no responses to a question from samples within a population. The number of positive responses in each of the samples is used to calculate the population percentage, which is the percentage of the entire population that would respond positively. This information is then organized into a box and whisker plot.

The 95% box and whisker plot is designed so that when further samples of the same size are taken from the population, 95 out of 100 samples will have the number of positive responses as indicated by the box in the plot. Five samples out of 100 will be outside of the box.

A box and whisker plot is one way of showing the amount of confidence that you have in predicting the outcome of a sample of a certain size.

5. A survey of parents was conducted to find out if they thought they were failures at parenting. The box and whisker plot charts that follow show the results of the survey. Study this chart and answer the questions that follow.

90% Box and Whisker Plots from Samples of Size 20

Proportion of Yeses in Sample
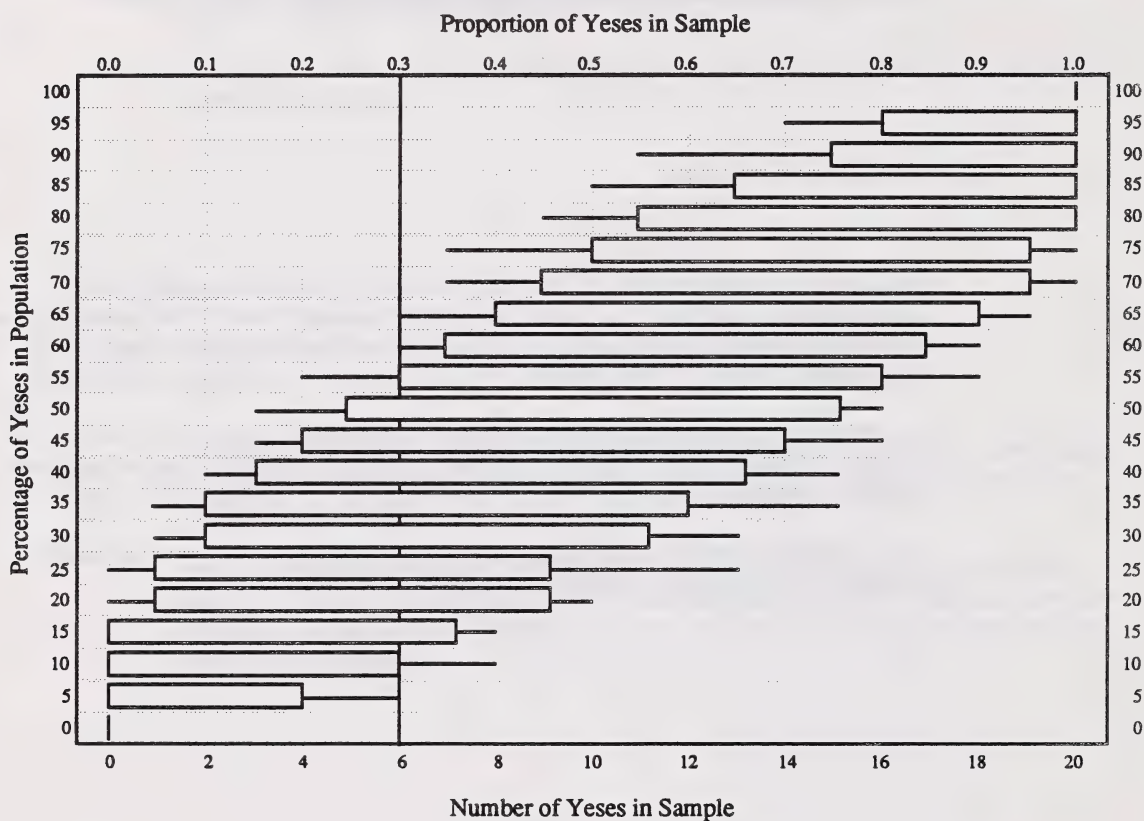
②   a.   How many parents answered yes?

          Six respondents said yes, they thought they were failures at parenting.

②   b.   How many parents were polled?

          The survey interviewed a random sample of twenty parents.

②   c.   What is the sample proportion?

          The sample proportion, according to the chart, is 0.3.

②   d.   What percentage of box and whisker plots was used?

          The surveyor used a 90% box and whisker plot from a sample size of 20.

②   e.   What is the confidence interval?

          According to the chart, 10% to 55% is a 90% confidence interval. In other words, between 10% and 55% would answer yes to the question 90% of the time.

②   f.   What are the lower and upper confidence limits?

          The lower confidence limit is 10% and the upper confidence limit is 55%.

⑤   6. Construct a sampling distribution using the following data.

Sample Number	Responses	Number of Yeses
1	NNYNNNYNNNYYNNNNNNN	4
2	YNNYNNNYYNNNNNYNNNNY	6
3	YYYNNNNNYNNNNYYNNNYNY	8
4	NNNYNNNNYYNNNNNNNYNN	4
5	YYYNNNNNYNYNNNNNNNYYY	8
6	NNNNNNNYYNNNYNYNNNNYN	5
7	YYYNNNYYNYYYNYNNNYNN	10
8	NNNYNYYNNNYYNNNNNYNN	6
9	NYNNNYYNNNYNNYNNNYYN	7
10	NNYNNNNYNNNYYNNNYYNY	7
11	NYNNNNNYYNNNNNYNNNYN	5
12	YYYNNNNNNNNNNYNYYYYNN	8
13	NNNYYYYYNNYNNYYNNNNY	9
14	YNNNNYYNNNYYNNNYNNNY	7
15	NNNNNYNYYYYNNYYYNNYN	9
16	NNNYNNNYNNNYYNNNYNYN	6
17	NNYNNNYNNNNYYNNNNNYY	6
18	NNNNNNYNNNYYYYNNNNYY	7
19	NNNNNYNNNNYYYNNYYNY	7
20	YYYNNNNNYNNNNNYNNNN	5

Number of Yeses	Sample Proportion	Frequency	Proportion of All Trials
0	0.00	0	0
1	0.05	0	0
2	0.10	0	0
3	0.15	0	0
4	0.20	2	0.1
5	0.25	3	0.15
6	0.30	4	0.2
7	0.35	5	0.25
8	0.40	3	0.15
9	0.45	2	0.1
10	0.50	1	0.05
11	0.55	0	0
12	0.60	0	0
13	0.65	0	0
14	0.70	0	0
15	0.75	0	0
16	0.80	0	0
17	0.85	0	0
18	0.90	0	0
19	0.95	0	0
20	1.00	0	0
Total		20	1.00

④ 7. Use the following sample distribution to make a 90% box and whisker plot.

Number of Yeses	Sample Proportion	Frequency	Proportion of All Trials
0	0.00	0	0
1	0.05	0	0
2	0.10	0	0
3	0.15	0	0
4	0.20	0	0
5	0.25	0	0
6	0.30	0	0
7	0.35	1	0.05
8	0.40	1	0.05
9	0.45	4	0.2
10	0.50	5	0.25
11	0.55	3	0.15
12	0.60	3	0.15
13	0.65	2	0.1
14	0.70	1	0.05
15	0.75	0	0
16	0.80	0	0
17	0.85	0	0
18	0.90	0	0
19	0.95	0	0
20	1.00	0	0
Total		20	1.00

Proportion of Yeses in Sample

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

0 2 4 6 8 10 12 14 16 18 20

Number of Yeses in Sample
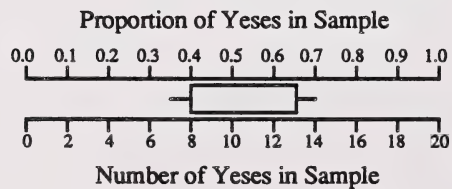
8. Fill in the following blanks.

(1)   a.   A __random sample__ is selected when each member of the population has an equal chance of being selected and the members of the sample are chosen independently.

(1)   b.   A sample is said to be __biased__ if a selection method tends to overrepresent or underrepresent some part of the population.

(1)   c.   Judgement and self-selected samples are both types of __convenience__ sampling.
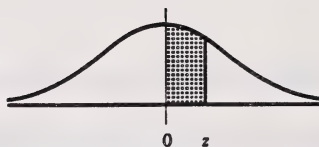
(1)   d.   A __stratified random__ sample occurs when the population is divided into different strata, and then a random sample is taken from each strata.

Topic 3

_____ marks

**Areas
under the
Standard
Normal Curve**

0   z

z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

MATHEMATICS 30

LEARNING FACILITATOR'S MANUAL

UNIT 7

Distance Learning

Alberta EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Topic 1 : Permutations

(3)

1. A theatre has seven gates. Two are on the north side, two are on the south side, and three are on the east side. In how many ways can a person enter and leave the theatre?

    A person can enter the theatre in seven ways and leave in seven ways. Therefore, the number of ways a person can enter and leave the theatre is $7 \times 7 = 49$ ways.

(4)

2. How many four-digit odd numbers larger than 8000 can be formed from the digits $0, 1, \ldots, 9$? (Digits can be repeated.)

2	10	10	5

The first digit can be 8 or 9. There are two choices for the first digit.
The last digit has to be an odd number. There are five choices since 1, 3, 5, 7, or 9 can be used.
The second and third digits both can be any one of the ten digits; thus, there are ten choices each.

The total number of ways $= 2 \times 10 \times 10 \times 5$

$= 1000$ ways

(4)

3. In how many ways can four letters be mailed if there are three mailboxes available?

    Each letter can be mailed in three different ways. (three mailboxes)
    Therefore, four letters can be mailed in $3 \times 3 \times 3 \times 3 = 3^4$

    $= 81$ ways.

(4)

4. Simplify the following:

   a. $\dfrac{(n-2)!}{(n-4)!2!}$

   $$\dfrac{(n-2)!}{(n-4)!2!} = \dfrac{(n-2)(n-3)(n-4)!}{(n-4)!2!}$$

   $$= \dfrac{(n-2)(n-3)}{2}$$

   b. $\dfrac{(n-3m-1)!3!}{(n-3m+1)!2!}$

   $$\dfrac{(n-3m-1)!3!}{(n-3m+1)!2!} = \dfrac{(n-3m-1)!(3)(2)!}{(n-3m+1)(n-3m)(n-3m-1)!2!}$$

   $$= \dfrac{3}{(n-3m+1)(n-3m)}$$

(2)

5. In how many different ways can eight different books be arranged?

   $8! = 40\ 320$ ways

(3)

6. Orasia has eleven different books, but her bookshelf has room for seven books only. In how many different ways can the books be placed on the bookshelf?

   There are eleven books. Orasia can choose any seven of them and arrange them on the shelf.

   $\therefore$ The number of arrangements $= {}_{11}P_7$

   $$= \dfrac{11!}{(11-7)!}$$

   $$= 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5$$

   $$= 1\ 663\ 200 \text{ ways}$$

④ 7. Peter has seven pictures, but no more than four of them can be hung in a row on the wall. In how many ways can this be done if at least one picture must be hung?

Peter can hang 1, 2, 3, or 4 pictures.
The number of ways $= {_7}P_1 + {_7}P_2 + {_7}P_3 + {_7}P_4$

$$= \frac{7!}{6!} + \frac{7!}{5!} + \frac{7!}{4!} + \frac{7!}{3!}$$
$$= 7 + 42 + 210 + 840$$
$$= 1099$$

④ 8. In how many ways can three cookies of the same kind and two chocolate bars of the same kind be distributed to five children if each child is to receive one item?

There are five items taken all at a time, of which three are alike and two others are alike.
The number of ways $= \frac{5!}{3!2!}$

$$= \frac{5 \times 4}{2 \times 1}$$
$$= 10 \text{ ways}$$

④ 9. A student has five English books and four French books. In how many ways can these books be arranged on a bookshelf so that the English books stand together and the French books stand together?

If the English books stand together, there are 5! ways to arrange them.
If the French books stand together, there are 4! ways to arrange them.
The English books and the French books represent two different groups of books; thus, they can be arranged in 2! ways.

The total number of ways to arrange these books is $5! \times 4! \times 2 = 5760$ ways.

⑤   10. In how many ways can fifteen different marbles be arranged in a circle if two of them must be placed together?

Two marbles must be placed together. They can be considered as one group. These two marbles can be arranged in 2! ways.
The remaining thirteen marbles together with the one group of two marbles can be arranged in $(14-1)!$ ways.
The total number of arrangements $= 2!(14-1)!$

$$= 2 \times 13! \text{ ways (or 12 454 041 600 ways)}$$

③   11. In how many ways can eleven different keys be strung together in a circle?

The number of ways $= \dfrac{(11-1)!}{2}$

$$= \dfrac{10!}{2}$$

$$= 1\ 814\ 400 \text{ ways}$$

⑤   12. Two cards are drawn one after the other randomly from an ordinary deck of 52 cards without replacement. What is the probability that these two cards are red kings?

There are two red kings in a deck of 52 cards. They can be arranged in 2! ways.

If any two cards are chosen, the number of arrangements is $_{52}P_2$.

The probability that the two cards are red kings $= \dfrac{2!}{_{52}P_2}$

$$= \dfrac{2!}{\left(\dfrac{52!}{50!}\right)}$$

$$= \dfrac{2!}{52 \times 51}$$

$$= \dfrac{1}{1326}$$

Topic 1

_____ marks

Topic 2 : Combinations

③ 1. Evaluate $\dfrac{_5C_3}{_5C_2}+_3C_1\times\dfrac{_3C_2}{_4C_4}-_7C_0$.

$$\dfrac{\dfrac{5!}{3!2!}}{\dfrac{5!}{2!3!}}+\dfrac{3!}{1!2!}\times\dfrac{\dfrac{3!}{2!1!}}{\dfrac{4!}{4!0!}}-\dfrac{7!}{7!0!}$$

$$=1+3\times\dfrac{3}{1}-1$$

$$=9$$

④ 2. There are ten different candies in a box. In how many ways can seven or more candies be selected?

You can select 7, 8, 9, or 10 candies.

The number of ways $=_{10}C_{10}+_{10}C_9+_{10}C_8+_{10}C_7$

$$=\dfrac{10!}{10!0!}+\dfrac{10!}{9!1!}+\dfrac{10!}{8!2!}+\dfrac{10!}{7!3!}$$

$$=1+10+45+120$$

$$=176$$

(4)

3. In how many ways can a group of twenty students be separated into two groups where each group contains ten students?

Ten students can be selected from the twenty students in $_{20}C_{10}$ ways.

Each time you select a group of ten students, you get another group of ten students.

$$\text{Therefore, the number of ways} = \frac{_{20}C_{10}}{2}$$
$$= \frac{\frac{20!}{10!10!}}{2}$$
$$= \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 11}{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 2}$$
$$= 92\ 378 \text{ ways}$$

(3)

4. In how many ways can George choose one or more of six different books?

$$\text{The number of ways} = 2^6 - 1$$
$$= 63 \text{ ways}$$

(4)

5. A committee of five students is to be chosen from a group of twenty students including the chairperson and the secretary of the students' union. If the chairperson of the students' union must be excluded and the secretary must be included, in how many ways can this be done?

Since the chairperson must be excluded and the secretary must be included, four students must be selected from the remaining eighteen students.

$$\text{The total number of selections} = {_1C_1} \times {_{18}C_4}$$
$$= 3060 \text{ ways}$$

(4)   6.   There are fifteen roses and twenty carnations. In how many ways can six flowers be selected so as to include at least one rose?

The number of ways in which any six flowers can be chosen is $_{35}C_6$.

The number of ways in which no rose is chosen is $_{20}C_6$.

The number of ways at least one rose is chosen $= {}_{35}C_6 - {}_{20}C_6$

$$= 1\ 623\ 160 - 38\ 760$$
$$= 1\ 584\ 400 \text{ ways}$$

(4)   7.   There are nine coplanar points. Four of them are collinear. No three of the other five are collinear. How many straight lines can be formed by joining any two of the nine points?

If no three of the nine points were collinear, the number of straight lines formed would be $_9C_2$.

If there were four noncollinear points, the number of straight lines formed by these four points would be $_4C_2$.

Four collinear points can form only one straight line.

The number of straight lines formed by joining any two of the nine points $= {}_9C_2 - {}_4C_2 + 1$

$$= \frac{9!}{7!2!} - \frac{4!}{2!2!} + 1$$
$$= 36 - 6 + 1$$
$$= 31 \text{ ways}$$

(4)   8.   There are nine quarters and five pennies in a box. Four coins are drawn at random. What is the probability that three of them are quarters?

The number of ways in which four coins can be drawn $= {}_{14}C_4$

$$= \frac{14!}{10!4!}$$
$$= 1001$$

The number of ways in which three quarters and one penny can be drawn $= {}_9C_3 \times {}_5C_1$

$$= \frac{9!}{6!3!} \times \frac{5!}{4!1!}$$
$$= 84 \times 5$$
$$= 420$$

The required probability is $P = \dfrac{420}{1001}$.

④ 9. Five oranges are chosen at random from twelve oranges of which four are sour. What is the probability that at least one of the chosen oranges is sour?

There are four sour oranges and eight sweet oranges.

The number of ways to choose all five sweet oranges is $_8C_5$.

The number of ways to choose any five oranges is $_{12}C_5$.

The probability that all five oranges are sweet is $\dfrac{_8C_5}{_{12}C_5}$.

The probability that at least one orange is sour $= 1 - \dfrac{_8C_5}{_{12}C_5}$

$$= 1 - \dfrac{\dfrac{8!}{5!3!}}{\dfrac{12!}{5!7!}}$$

$$= 1 - \dfrac{56}{792}$$

$$= \dfrac{736}{792}$$

$$= \dfrac{92}{99}$$

③ 10. Use the binomial theorem to expand $\left(x^2 - 2y\right)^6$.

$$\left(x^2\right)^6 - 6\left(x^2\right)^5(2y) + 15\left(x^2\right)^4(2y)^2 - 20\left(x^2\right)^3(2y)^3 + 15\left(x^2\right)^2(2y)^4 - 6\left(x^2\right)(2y)^5 + (2y)^6$$

$$= x^{12} - 12x^{10}y + 60x^8y^2 - 160x^6y^3 + 240x^4y^4 - 192x^2y^5 + 64y^6$$

④   11. Determine the twelfth term in the expansion of $\left(x^2-\frac{1}{2}\right)^{16}$.

The rth term is $_nC_{r-1}\,a^{n-r+1}\,b^{r-1}$.

$n=16,\ r=12,\ a=x^2,\ b=\left(-\frac{1}{2}\right)$

$$t_{12}=\,_{16}C_{12-1}\left(x^2\right)^{16-12+1}\left(-\frac{1}{2}\right)^{12-1}$$

$$=\frac{16!}{11!5!}\left(x^2\right)^5\left(-\frac{1}{2}\right)^{11}$$

$$=4368x^{10}\times\frac{-1}{2^{11}}$$

$$=\frac{-4368}{2048}x^{10}$$

$$=-2\frac{17}{128}x^{10}$$

④   12. Determine the value of r if $_{13}C_{r-1}\,x^{10}\,y^{3r-3}$ is a term in the expansion of $\left(x^2+y^3\right)^{13}$.

The rth term is $_nC_{r-1}\,a^{n-r+1}\,b^{r-1}$, where $n=13$, $a=x^2$, and $b=y^3$.

Compare $_{13}C_{r-1}\,x^{10}\,y^{3r-3}$ to $_{13}C_{r-1}\left(x^2\right)^{n-r+1}\left(y^3\right)^{r-1}$.

Thus, $10=2(n-r+1)$

$\qquad\quad =2(13-r+1)$

$\qquad\quad =26-2r+2$

$\qquad\quad =28-2r$

$\quad 2r=28-10$

$\qquad\quad =18$

$\quad\ r=9$

(3)

13. Determine the numerical coefficient of the seventh term in the expansion $\left(x^2-3\right)^{11}$.

$$t_r = {}_nC_{r-1}\, a^{n-r+1}\, b^{r-1}$$
$$t_7 = {}_{11}C_6\left(x^2\right)^5(-3)^6$$
$$= \frac{11!}{6!5!} x^{10}(729)$$
$$= 336\ 798 x^{10}$$

Thus, the numerical coefficient is 336 798.

(2)

14. Use Pascal's theorem to find an equivalent expression of ${}_{18}C_9 + {}_{18}C_{10}$ in the form ${}_nC_r$.

Pascal's theorem: ${}_nC_r + {}_nC_{r+1} = {}_{n+1}C_{r+1}$
$${}_{18}C_9 + {}_{18}C_{9+1} = {}_{18+1}C_{9+1}$$
Thus, ${}_{18}C_9 + {}_{18}C_{10} = {}_{19}C_{10}$.

⑤ 15. There are five different stamps from Australia, six different stamps from Japan, and seven different stamps from the United States in an envelope. Two stamps are drawn at random from the envelope. What is the probability of drawing one stamp each from Australia and Japan or one stamp each from Japan and the United States?

There are $5 + 6 + 7 = 18$ stamps.

The number of combinations of eighteen stamps taken two at a time $= {}_{18}C_2$

$$= \frac{18!}{2!16!}$$
$$= \frac{18 \times 17}{2}$$
$$= 153$$

The number of combinations of one stamp from Australia and one stamp from Japan
$= {}_5C_1 \times {}_6C_1$
$= 5 \times 6$
$= 30$

The number of combinations of one stamp from Japan and one stamp from the United States
$= {}_6C_1 \times {}_7C_1$
$= 6 \times 7$
$= 42$

The required probability $= \dfrac{30}{153} + \dfrac{42}{153}$

$$= \frac{72}{153}$$
$$= \frac{8}{17}$$

Topic 2

_____ marks

Final Test

There are two copies of the final test: the student's copy which is perforated and designed for photocopying and possible faxing and the teacher's copy which includes a marking guide.

Note:

The student's copy of this final test should be kept secure by the teacher. Students should not have access to this test until it is assigned in a supervised situation. The answers should be stored securely and retained by the teacher at all times.

MATHEMATICS 30

FINAL TEST

GENERAL INSTRUCTIONS

1. The time limit for this test is **$2\frac{1}{2}$ hours.**

2. There are **120 marks** on this test.

    PART A : Multiple Choice   - 82 marks

    PART B : Open-ended Scannable Questions   - 10 marks

    PART C : Written Response   - 28 marks

3. Any hand-held calculator may be used. Calculators having graphing capabilities, built-in formulas, mathematical functions, or other programmable features are allowed.

4. All necessary tables and a list of formulas have been provided at the end of the test.

5. It is better to complete all work which you know well first. After that you can deal with the difficulties. Do not spend too much time on any one problem. If you finish the test before the $2\frac{1}{2}$ hour time limit, go back and check you answers.

PART A: MULTIPLE CHOICE

All multiple-choice questions must be answered on the Part A Response Page included in your test.

Read each question carefully and decide which of the choices BEST completes the statement or answers the question. Locate the question number on the Response Page and place the appropriate letter A, B, C, or D in the blank.

Example:

83.   The value of 2^3 is   Response Page

     A.   6   __B__   83.

     B.   8

     C.   – 8

     D.   – 6

1.   From the top of a 150 m cliff, the angle of depression to a boat in the harbour below is 25°. How far is the boat from the base of the cliff? Round your answer to the nearest metre.

     A.   70 m

     B.   322 m

     C.   63 m

     D.   355 m

TEACHER'S COPY

2. Cos 75° is equal to

    A. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

    B. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

    C. $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

    D. $\dfrac{\sqrt{6}-\sqrt{2}}{8}$

3. The solution set of $4\cos^2\theta = 3,\, 0 \le \theta \le 2\pi$ is

    A. $\left\{\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}\right\}$

    B. $\left\{\dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{5\pi}{6}\right\}$

    C. $\left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right\}$

    D. $\left\{\dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{7\pi}{12}, \dfrac{11\pi}{12}\right\}$

4. The value of sin 240° is equal to the value of

    A. $-\sin 30°$

    B. $-\sin 60°$

    C. $\cos 30°$

    D. $-\cos 60°$

TEACHER'S COPY

5. If $\cos \theta = -\dfrac{8}{17}$ and $\sin \theta$ is positive, then $\tan \theta$ equals

   A. $-\dfrac{15}{17}$

   B. $-\dfrac{17}{15}$

   C. $-\dfrac{8}{15}$

   D. $-\dfrac{15}{8}$

6. For the function $y = -3\sin(2\theta)$, the period is

   A. π

   B. 2π

   C. 3π

   D. 4π

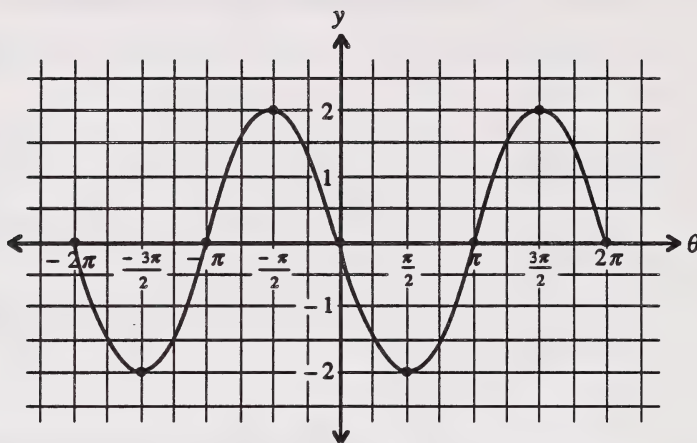7. If $\theta \neq n\pi$, $n \in I$, $\sec^2\theta - \tan^2\theta - \cos^2\theta$ is equal to

   A. $1 + \tan^2\theta$

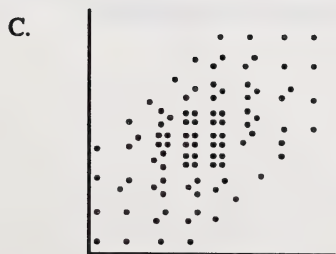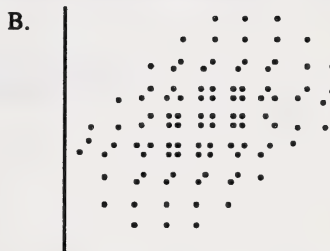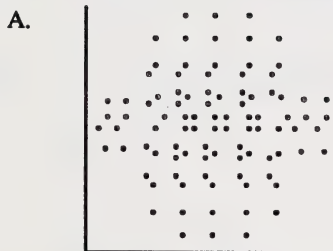   B. $\csc^2\theta$

   C. $\sin^2\theta$

   D. $-\cos^2\theta$

TEACHER'S COPY

8.

The graph shown represents the function

A. $y = 2\sin\left(\theta + \dfrac{\pi}{2}\right)$ $-2\pi \le \theta \le 2\pi$

B. $y = 2\cos\left(\theta + \dfrac{\pi}{2}\right)$ $-2\pi \le \theta \le 2\pi$

C. $y = -2\sin\left(\theta - \dfrac{\pi}{2}\right)$ $-2\pi \le \theta \le 2\pi$

D. $y = -2\cos\left(\theta - \dfrac{\pi}{2}\right)$ $-2\pi \le \theta \le 2\pi$

TEACHER'S COPY

9. On a certain cold day, the mean waiting time for a taxi was 40 minutes, with a standard deviation of 12 minutes. Of 50 callers, the number who received a taxi within 30 minutes was

   A. 10

   B. 20

   C. 30

   D. 40

10. Refrigerator manufacturers determine the mean life of their refrigerator motors to be 11 years, with a standard deviation of 4 years. If they guarantee their refrigerator motors for 3 years, the percentage of motors that will have to be replaced is

    A. 47.72%

    B. 0.47%

    C. 0.27%

    D. 2.28%

11. Which of the following graphs shows a strong positive correlation?

A.

B.

C.

D.

TEACHER'S COPY

12. For a normal distribution, the mean (μ) is 15, and the standard deviation (σ) is 6. The probability to four decimal places of x for the interval $10.2 \leq x \leq 12.8$ is

   A.   0.4287

   B.   0.6436

   C.   0.1438

   D.   0.2664

13. Assuming the sequence $-7, -4, -1, 2, 5, 8, \ldots$ is arithmetic, the 27th term is

   A.   71

   B.   74

   C.   81

   D.   78

14. The sum of n terms in a geometric series is 171. If the first term is 1 and the common ratio is -2, then the number of terms in the series is

   A.   11

   B.   9

   C.   7

   D.   5

15. In an arithmetic series the sum of n terms is 788.5. The first and last terms in this series are 10 and 73 respectively. The number of terms in the series is

   A   12.

   B.   13

   C.   18

   D.   19

TEACHER'S COPY

16. For the geometric series $3+3\sqrt{3}+9+\ldots$, the sum of the first seven terms is

    A. $120+39\sqrt{3}$

    B. $120-39\sqrt{3}$

    C. $240+39\sqrt{3}$

    D. $240-39\sqrt{3}$

17. Using sigma notation, the series
    $(2 \times 5) + (4 \times 7) + (6 \times 9) + \ldots + (12 \times 15)$ is

    A. $\displaystyle\sum_{k=1}^{6}(k)(k+3)$

    B. $\displaystyle\sum_{k=1}^{6}(2k)(2k+3)$

    C. $\displaystyle\sum_{k=0}^{4}(k+2)(k+5)$

    D. $\displaystyle\sum_{k=1}^{6}(k+2)(2k+3)$

18. If $2^x = 5$, then $\log_2(25)$ in terms of x is

    A. 10^x

    B. 3^{5x}

    C. x^5

    D. $2x$

TEACHER'S COPY

19. $\dfrac{\log d}{h} + \dfrac{m\log t}{h}$ is equal to

    A. $\log\sqrt[h]{d \times mt}$

    B. $\log\sqrt[h]{\dfrac{t^m}{d}}$

    C. $\log\sqrt[h]{dt^m}$

    D. $\log dt^m - h$

20. The value of x in the equation $2^x = 3^{x+1}$ is

    A. -2.709

    B. 3.8

    C. 0.602

    D. 0.48

21. If $\log 4x - \log\dfrac{x}{2} = y$, then its exponential form is

    A. $10^y = 8$

    B. $8^y = 10$

    C. $y^8 = 10$

    D. $y^{10} = 8$

TEACHER'S COPY

Use the information below to answer question 22.

$$N(t) = N_0\left(2^{\frac{t}{40}}\right), \text{ where } N(t) = \text{final number of bacteria,}$$
$$N_0 = \text{initial number of bacteria,}$$
$$\text{and} \quad t = \text{time in minutes.}$$

22. The logarithmic expression for the time (t) it takes the number of bacteria to increase from 10 000 to 400 000 is

   A. $\dfrac{\log_2 40}{40}$

   B. $\log_2\left(2^{\frac{1}{40}}\right)$

   C. $\dfrac{40}{\log_2 40}$

   D. $\dfrac{40 \log 40}{\log 2}$

23. The equation of a circle with centre $(0, 0)$ and a y-intercept of -3 is

   A. $x^2 + (y-3)^2 = 0$

   B. $(x-3)^2 + y^2 = 0$

   C. $x^2 + y^2 = 9$

   D. $x^2 + (y-3)^2 = 9$

24. A circle with centre $(-2, 3)$ and tangent to the line $y = 0$ is defined by the equation

   A. $(x+2)^2 + (y-3)^2 = 9$

   B. $(x-2)^2 + (y+3)^2 = 9$

   C. $(x+2)^2 + (y-3)^2 = 4$

   D. $(x-2)^2 + (y+3)^2 = 4$

TEACHER'S COPY

25. If the equation of a parabola is $y^2 = -8x$, then the focus and directrix respectively are

    A. $F(2, 0), \quad x = 2$

    B. $F(2, 0), \quad x = -2$

    C. $F(-2, 0), \quad x = 2$

    D. $F(-2, 0), \quad x = -2$

26. What is the equation of a parabola for which the vertex is (4, 2) and the focus is 3 units to the left of the vertex?

    A. $(x-4)^2 = -12(y-2)$

    B. $(y-2)^2 = 12(x-4)$

    C. $(y-2)^2 = -12(x-4)$

    D. $(x+4)^2 = 12(y+2)$

27. The equation of a circle is $x^2 + y^2 - 4x + 6y - 3 = 0$.
The centre and radius respectively of this circle are

    A. $(-2, 3), \quad 16$

    B. $(2, -3), \quad 16$

    C. $(-2, 3), \quad 4$

    D. $(2, -3), \quad 4$

28. If the equation of the ellipse is $25x^2 + 16y^2 = 400$, then the length of the major axis is

    A.   4

    B.   5

    C.   8

    D.   10

TEACHER'S COPY

29. The equation of the ellipse centred at the origin with a major axis of 18 units and one focus at $(0, -7)$ is

    A. $\dfrac{x^2}{81} + \dfrac{y^2}{32} = 1$

    B. $\dfrac{x^2}{81} - \dfrac{y^2}{32} = 1$

    C. $\dfrac{x^2}{32} + \dfrac{y^2}{81} = 1$

    D. $\dfrac{x^2}{81} + \dfrac{y^2}{49} = 1$

30. A point (x, y) moves so that the difference between its distances from $(0, 7)$ and $(0, -7)$ is always 10. One possible value of x when y equals 6 is

    A. $\dfrac{\sqrt{264}}{5}$

    B. $\sqrt{\dfrac{300}{24}}$

    C. $\dfrac{\sqrt{1464}}{5}$

    D. $\sqrt{\dfrac{1500}{24}}$

31. The length of the conjugate axis of a hyperbola whose centre is at the origin is 24, and the length of the transverse axis is 10. How far is the focus from the origin?

    A.   11

    B.   13

    C.   22

    D.   26

TEACHER'S COPY

32. The equation of the hyperbola with centre at (0, 0), one focus at (0, 5), and one vertex at (0, − 2) is

A. $\dfrac{x^2}{21} - \dfrac{y^2}{4} = 1$

B. $\dfrac{y^2}{21} - \dfrac{x^2}{4} = 1$

C. $\dfrac{x^2}{4} - \dfrac{y^2}{21} = 1$
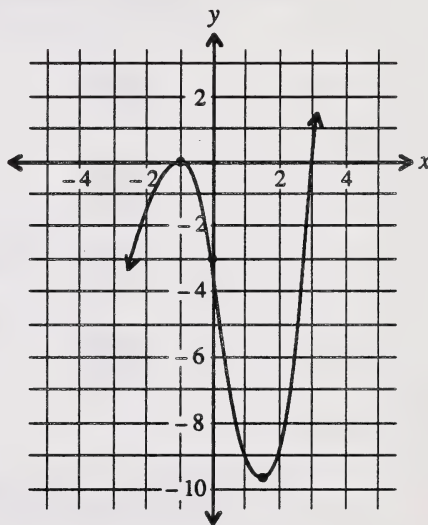
D. $\dfrac{y^2}{4} + \dfrac{x^2}{21} = 1$

33. The graph to the right is the graph of one of the functions below. The function is

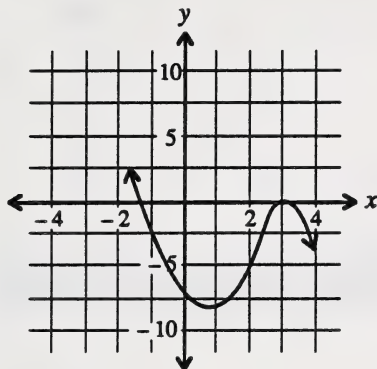A. $x^3 - x^2 - 5x - 3$

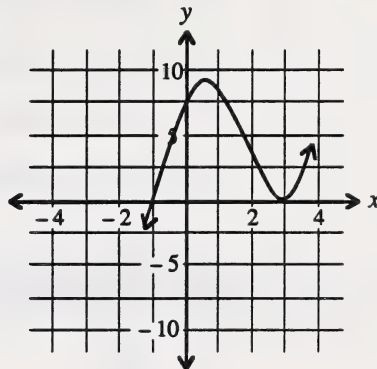B. $x^3 - x^2 - 4x - 6$

C. $x^3 + x^2 - 5x + 3$

D. $x^3 - 5x^2 + 3x + 9$

TEACHER'S COPY

34. If $y = x^3 - 5x^2 + 3x + 9$, then the sketch that best represents the graph is

A.

B.

C.

D.

35. The coefficient of x^3 in a third-degree polynomial $P(x)$ is 2. If $P(2) = P(-2) = P(3) = 0$, then $P(x)$ is equal to

A. $2x^3 + 2x^2 + 8x - 24$

B. $2x^3 - 6x^2 - 8x + 24$

C. $2x^3 - 2x^2 - 8x - 24$

D. $2x^3 + 14x^2 + 8x - 24$

TEACHER'S COPY

36. When $3x^3-4x^2+5x-6$ is divided by $x-1$, the quotient and remainder are

   A. $3x^2-x+4$ and -2

   B. $3x^2-7x+12$ and -18

   C. $3x^2-x+6$ and 0

   D. $3x^2+7x+12$ and -18

37. In how many ways can four different chemistry books and six different biology books be arranged on a shelf if the chemistry books are always kept together?

   A.   2880

   B.   17 280

   C.   120 960

   D.   967 680

38. In how many ways can eight people be seated at a round table if three of them must always sit together?

   A.   720

   B.   1140

   C.   4320

   D.   5040

39. How many different signals can be made with five different flags by raising 1, 2, 3 or 4 flags at a time on a flagpole? (No flags is not considered as a signal.)

   A.   320

   B.   205

   C.   30

   D.   26

TEACHER'S COPY

40. A committee of four is to be selected from seven men and eight women. How many different committees can be formed if there must be at least one woman?

    A. 960

    B. 1130

    C. 1330

    D. 1560

41. There are five girls and three boys in a class. If a group of three students is chosen at random to form a committee, determine the probability that two of them are boys.

    A. $\dfrac{17}{56}$

    B. $\dfrac{15}{56}$

    C. $\dfrac{13}{56}$

    D. $\dfrac{11}{56}$

PART A: RESPONSE PAGE

B	1.	**D**	11.	**A**	21.	**B**	31.	**B**	41.
A	2.	**C**	12.	**D**	22.	**D**	32.		
C	3.	**A**	13.	**C**	23.	**A**	33.		
B	4.	**B**	14.	**C**	24.	**B**	34.		
D	5.	**D**	15.	**C**	25.	**B**	35.		
A	6.	**A**	16.	**C**	26.	**A**	36.		
C	7.	**B**	17.	**D**	27.	**C**	37.		
B	8.	**D**	18.	**D**	28.	**A**	38.		
A	9.	**C**	19.	**C**	29.	**B**	39.		
D	10.	**A**	20.	**A**	30.	**C**	40.		

TEACHER'S COPY

PART B: OPEN-ENDED SCANNABLE QUESTIONS

Record your answer next to each question by writing it in the corresponding answer field and by filling in one circle in EVERY column as illustrated. If you wish to change an answer, please erase your first answer completely.

Examples:

a. If 10, x, 90 are consecutive terms of a geometric sequence, then a positive value of x to the nearest tenth is _____.

$$\frac{x}{10} = \frac{90}{x}$$

$$x^2 = 900$$

$$x = \pm 30$$

Record 030.0.

$$3, \ 9, \ 27, \ldots, \ 3^n, \ldots$$

b. If $\cos \theta = 0.7821$ where $0° \le \theta \le 90°$, then the measure of θ correct to the nearest tenth of a degree is _____ .

$\theta = 38.546\ 746\ 41$
Record 038.5.

TEACHER'S COPY

Value

(2) 1. It is assumed that the science test scores are normally distributed with $\mu = 80$ and $\sigma = 25$. The probability, to the nearest tenth, that a student's score will be more than 87 is

_____ .

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{87 - 80}{25}$$

$$= 0.28$$

$$P = 0.1103$$

Score more than 87 is

$0.5000 - 0.1103$

$= 0.3897$

$\doteq 000.4$

0	0	0	4

(2) 2. In a geometric sequence, the first term is 10 and the fourth term is 80. The seventh term, correct to the nearest tenth, is _____ .

$$t_n = ar^{n-1}$$

$$80 = 10r^{4-1}$$

$$8 = r^3$$

$$2 = r$$

$$t_7 = 10 \times 2^6$$

$$= 10 \times 64$$

$$= 640.0$$

6	4	0	0

TEACHER'S COPY

Value

②

3. If the remainder is 43 when $2x^3 - 3x^2 + 4x + m$ is divided by $x - 3$, m, correct to the nearest tenth, is _____ .

$$P(3) = 2(3)^3 - 3(3)^2 + 4(3) + m$$
$$= 54 - 27 + 12 + m$$
$$= 39 + m$$
$$39 + m = 43$$
$$m = 4$$

0	0	4	0
●	●	⊙.	●
①	①	①.	①
②	②	②.	②
③	③	③.	③
④	④	●.	④
⑤	⑤	⑤.	⑤
⑥	⑥	⑥.	⑥
⑦	⑦	⑦.	⑦
⑧	⑧	⑧.	⑧
⑨	⑨	⑨.	⑨

②

4. If $\log_{32}(x) = -\frac{1}{5}$, then the value of x correct to the nearest tenth is _____ .

$$(32)^{-\frac{1}{5}} = x$$
$$\left(2^5\right)^{-\frac{1}{5}} = x$$
$$2^{-1} = x$$
$$x = \frac{1}{2} = 0.5$$

0	0	0	5
●	●	●.	⓪
①	①	①.	①
②	②	②.	②
③	③	③.	③
④	④	④.	④
⑤	⑤	⑤.	●
⑥	⑥	⑥.	⑥
⑦	⑦	⑦.	⑦
⑧	⑧	⑧.	⑧
⑨	⑨	⑨.	⑨

TEACHER'S COPY

Value

②

5. A pendulum 50 cm long swings through an angle of 36°. What is the length of the arc, correct to the nearest tenth of a centimetre, through which a spider swings if it clings to the pendulum 45 cm from its top?

$$a = r\theta$$
$$= 45\left(\frac{36}{180}\pi\right)$$
$$= 28.3 \text{ cm}$$

TEACHER'S COPY

Value

PART C: WRITTEN RESPONSE

Be sure to show all your work. Part marks will be given for partially correct solutions.

(4)
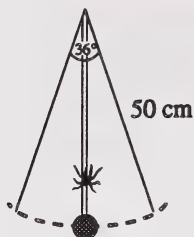
1. In a geometric series, the sum of the first five terms is -7320 and the sum of the first six terms is $21\,840$. The fourth term is 3240. Calculate S_9 for the series.

$S_5 = -7320,\ S_6 = 21\,840,\ t_4 = 3240$

$$S_5 + t_6 = S_6$$
$$-7320 + t_6 = 21\,840$$
$$t_6 = 29\,160$$
$$t_6 = ar^5 = 29\,160$$
$$t_4 = ar^3 = 3240$$
$$\frac{ar^5}{ar^3} = \frac{29\,160}{3240}$$
$$r^2 = 9$$
$$r = \pm 3 \ \ (3 \text{ will not work}).$$

$$ar^3 = 3240$$
$$a(-3)^3 = 3240$$
$$a = -120$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$
$$S_9 = \frac{-120\left[(-3)^9 - 1\right]}{-3 - 1}$$
$$= 30[-19\,683 - 1]$$
$$= -590\,520$$

TEACHER'S COPY

Value

④ 2. Express the polynomial $3x^4 - 5x^3 - 5x^2 + 5x + 2$ as a product of first-degree factors.

Divisors of 2: ±1, ±2

Divisors of 3: ±1, ±3

Potential zeros: $\pm 1, \pm\frac{2}{3}, \pm\frac{1}{3}, \pm 2$

$$
\begin{array}{r|rrrrr}
1 & 3 & -5 & -5 & 5 & 2 \\
  &   & 3 & -8 & 3 & 2 \\
\hline
-1 & 3 & -8 & 3 & 2 & 0 \\
  &   & -3 & 5 & 2 & 0 \\
\hline
  & 3 & -5 & -2 & 0 & 0
\end{array}
$$

$\therefore x + 1$ is a factor.

$\therefore x - 1$ is a factor.

The factors are $(x+1)(x-1)(3x^2 - 5x - 2)$

$= (x+1)(x-1)(3x+1)(x-2)$.

TEACHER'S COPY

Value

(4)

3. Determine the sixth term in the expansion of $(2x-3y)^9$.

The rth term in the expansion of $(a+b)^n$ is $_nC_{r-1}\,a^{n-r+1}\,b^{r-1}$.

$n=9,\ r=6,\ a=2x,\ b=(-3y)$

$t_6 = {}_9C_{6-1}(2x)^{9-6+1}(-3y)^{6-1}$

$\quad = {}_9C_5(2x)^4(-3y)^5$

$\quad = \dfrac{9!}{5!4!}\left(16x^4\right)\left(-243y^5\right)$

$\quad = -489\,888x^4y^5$

(4)

4. The roof of an arena is in the shape of one half of an ellipse. Use the dimensions shown to find the equation of the ellipse.
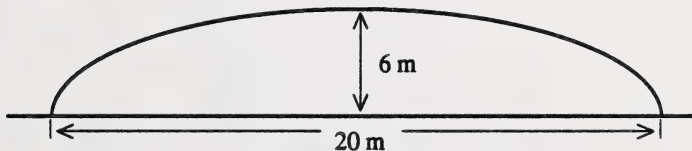
$2a = 20$

$a = 10$

$b = 6$

$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

$\dfrac{x^2}{100}+\dfrac{y^2}{36}=1$

TEACHER'S COPY

Value

④

5. An I.Q. test was given to a group of students. The result follows a normal distribution with a mean of 120 and a standard deviation of 15. Between what two values on either side of the mean will 10.9% of the scores be found?

$$\mu = 120 \qquad\qquad \sigma = 15$$

$$10.9\% \div 2 = 5.45\%$$

$$= 0.0545$$

$$z = 0.14 \text{ or } -0.14$$

$$\pm 0.14 = \frac{x - 120}{15}$$

$$\pm 2.10 = x - 120$$

$$x = 122.1, \ 117.9$$

The scores are between 117.9 and 122.1.

TEACHER'S COPY

Value

(4)

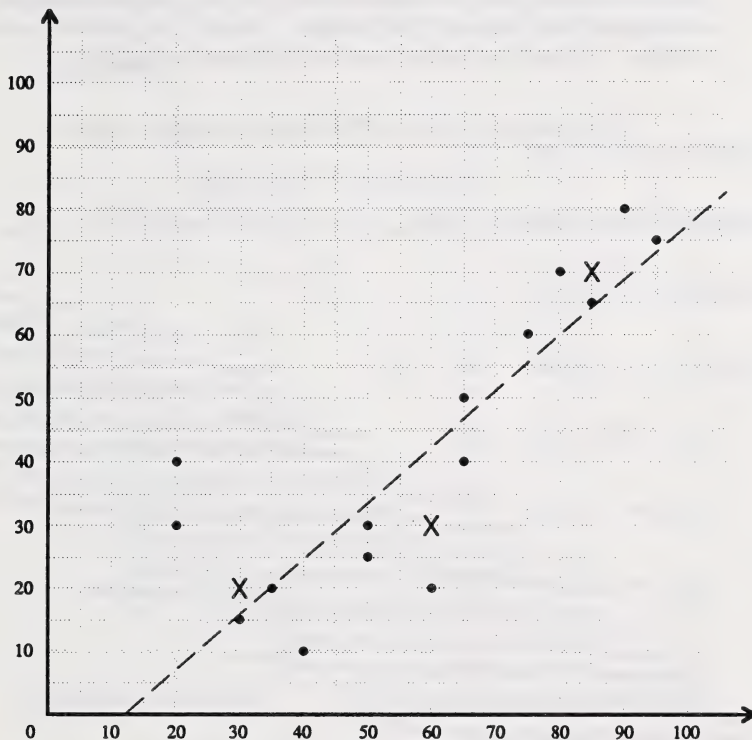6. Draw the line of best fit for the following scatterplot using the median fit method.

The first strip points are: (20, 30), (20, 40), (30, 15), (35, 20), (40, 10).
The median x-coordinate is 30.   20, 20, 30, 35, 40

The median y-coordinate is 20.   10, 15, 20, 30. 40

     Plot an x at point (30, 20).
The second strip points are: (50, 25), (50, 30), (60, 20), (65, 40), (65, 50).
The median x-coordinate is 60.   50, 50, 60, 65, 65

The median y-coordinate is 30.   20, 25, 30, 40, 50

     Plot an x at point (60, 30).
The third stip points are: (75, 60), (80, 70), (85, 65), (90, 80), (95, 75).
The median x-coordinate is 85.   75, 80, 85, 90, 95

The median y-coordinate is 70.   60, 65, 70, 75, 80

     Plot an x at point (85, 70).
Place a ruler on the median points of the two outside strips. Slide the ruler parallel to itself, one third of the distance to the median point in the middle strip. Draw a straight line.

TEACHER'S COPY

Value

(4)

7. An environmental group conducted a survey of forty people to find out if they thought that atmospheric pollution was on the increase. Twenty four persons responded yes to the question. Use the chart below to answer the questions that follow.

90% Box and Whisker Plots from Samples of Size 40

Proportion of Yeses in Sample

Number of Yeses in Sample
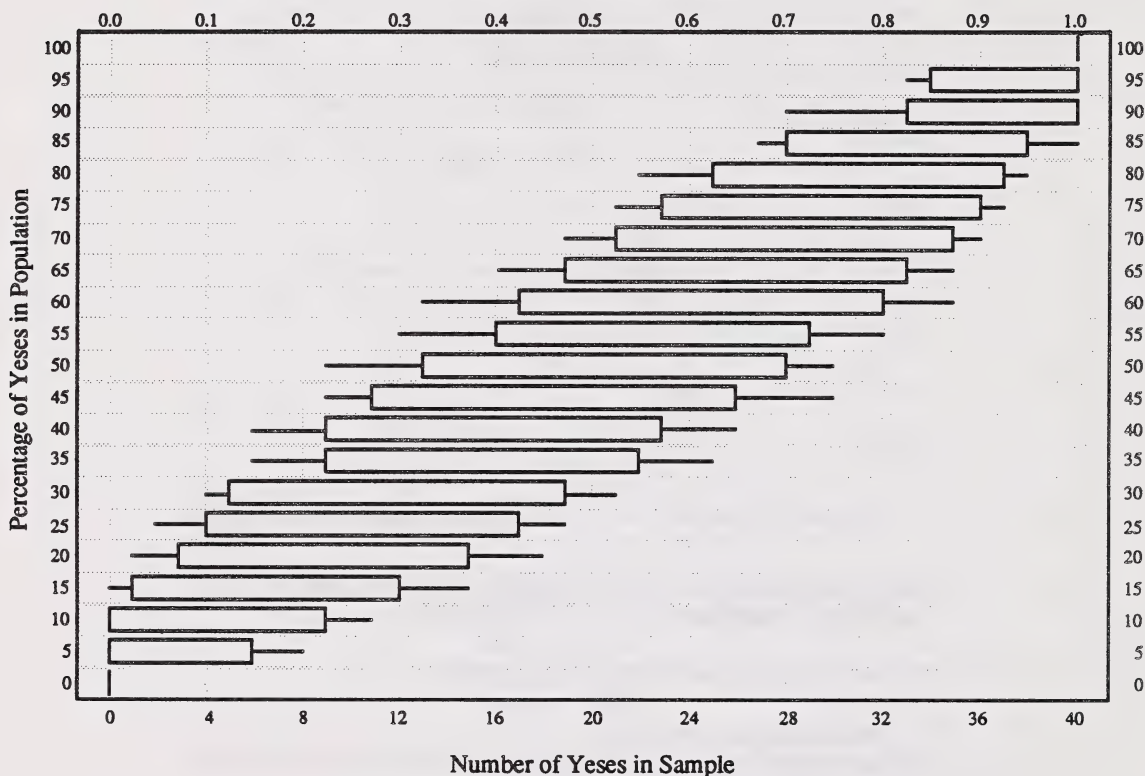
a. What is the confidence interval?

The 90% confidence interval is 45% to 75%.

b. Explain what is meant by the confidence interval in a.

If the environmental group surveyed a sample of forty people, they would be 90% certain that 45% to 75% of the people would say yes.

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MATHEMATICS 30
FORMULA SHEET

A.   Trigonometry

1. $\pi = 3.14159$

2. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

3. $a^2 = b^2 + c^2 - 2bc \cos A$

4. $\sin^2 A + \cos^2 A = 1$

5. $1 + \tan^2 A = \sec^2 A$

6. $1 + \cot^2 A = \csc^2 A$

7. $\sin\left(\dfrac{\pi}{2} - \theta\right) = \cos \theta$

8. $\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin \theta$

9. $\sin (A + B) = \sin A \cos B + \cos A \sin B$

10. $\sin (A - B) = \sin A \cos B - \cos A \sin B$

11. $\cos (A + B) = \cos A \cos B - \sin A \sin B$

12. $\cos (A - B) = \cos A \cos B + \sin A \sin B$

13. $\sin (-\theta) = -\sin \theta$

14. $\cos (-\theta) = \cos \theta$

15. $\tan (-\theta) = -\tan \theta$

B.   Quadratic Relations

1. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

3. $M\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

4. $(x - h)^2 + (y - k)^2 = r^2$

5. $x^2 + y^2 + Dx + Ey + F = 0$

6. $(y - k)^2 = 4p(x - h)$

7. $(x - h)^2 = 4p(y - k)$

8. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, a^2 = b^2 + c^2$

9. $\dfrac{y^2}{a^2} + \dfrac{x^2}{b^2} = 1, a^2 = b^2 + c^2$

10. $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1, c^2 = a^2 + b^2$

11. $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1, c^2 = a^2 + b^2$

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C. Sequences, Series, and Limits

1. $t_n = a + (n-1)d$

2. $S_n = \dfrac{n(a+t_n)}{2}$

3. $S_n = \dfrac{n[2a+(n-1)d]}{2}$

4. $A = P(1+i)^n$

5. $t_n = ar^{n-1}$

6. $S_n = \dfrac{a(r^n - 1)}{r-1}$

7. $S_n = \dfrac{rt_n - a}{r-1}$

8. $S = \dfrac{a}{1-r}, -1 < r < 1$

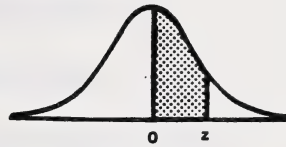## D. Statistics

1. $\mu = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$

2. $\sigma = \sqrt{\dfrac{(x_1 - \mu)^2 + \cdots + (x_n - \mu)^2}{n}}$

3. $z = \dfrac{x - \mu}{\sigma}$

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Areas under the Standard Normal Curve

z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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MATHEMATICS 30

FINAL TEST

GENERAL INSTRUCTIONS

1. The time limit for this test is **$2\frac{1}{2}$ hours.**

2. There are **120 marks** on this test.

    PART A : Multiple Choice   - 82 marks

    PART B : Open-ended Scannable Questions   - 10 marks

    PART C : Written Response   - 28 marks

3. Any hand-held calculator may be used. Calculators having graphing capabilities, built-in formulas, mathematical functions, or other programmable features are allowed.

4. All necessary tables and a list of formulas have been provided at the end of the test.

5. It is better to complete all work which you know well first. After that you can deal with the difficulties. Do not spend too much time on any one problem. If you finish the test before the $2\frac{1}{2}$ hour time limit, go back and check you answers.

PART A: MULTIPLE CHOICE

All multiple-choice questions must be answered on the Part A Response Page included in your test.

Read each question carefully and decide which of the choices BEST completes the statement or answers the question. Locate the question number on the Response Page and place the appropriate letter A, B, C, or D in the blank.

Example:

83.   The value of 2^3 is

  A.   6

  B.   8

  C. -8

  D. -6

Response Page

  __B__   83.

1.   From the top of a 150 m cliff, the angle of depression to a boat in the harbour below is 25°. How far is the boat from the base of the cliff? Round your answer to the nearest metre.

  A.   70 m

  B.   322 m

  C.   63 m

  D.   355 m

2. Cos 75° is equal to

   A. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

   B. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

   C. $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

   D. $\dfrac{\sqrt{6}-\sqrt{2}}{8}$

3. The solution set of $4\cos^2\theta = 3, 0 \le \theta \le 2\pi$ is

   A. $\left\{ \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3} \right\}$

   B. $\left\{ \dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{5\pi}{6} \right\}$

   C. $\left\{ \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6} \right\}$

   D. $\left\{ \dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{7\pi}{12}, \dfrac{11\pi}{12} \right\}$

4. The value of sin 240° is equal to the value of

   A. $-\sin 30°$

   B. $-\sin 60°$

   C. $\cos 30°$

   D. $-\cos 60°$

5. If $\cos \theta = -\dfrac{8}{17}$ and $\sin \theta$ is positive, then $\tan \theta$ equals

   A. $-\dfrac{15}{17}$

   B. $-\dfrac{17}{15}$

   C. $-\dfrac{8}{15}$

   D. $-\dfrac{15}{8}$

6. For the function $y = -3\sin(2\theta)$, the period is

   A. π

   B. 2π

   C. 3π

   D. 4π

7. If $\theta \neq n\pi$, $n \in I$, $\sec^2 \theta - \tan^2 \theta - \cos^2 \theta$ is equal to
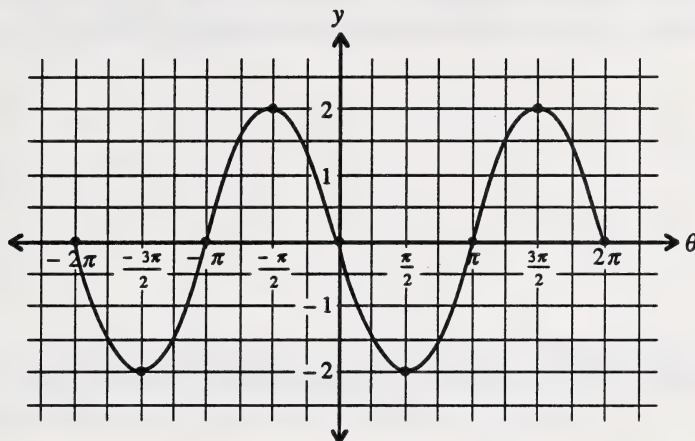
   A. $1 + \tan^2 \theta$

   B. $\csc^2 \theta$

   C. $\sin^2 \theta$

   D. $-\cos^2 \theta$

8.

The graph shown represents the function

A. $y = 2\sin\left(\theta + \dfrac{\pi}{2}\right)$ $\qquad -2\pi \le \theta \le 2\pi$
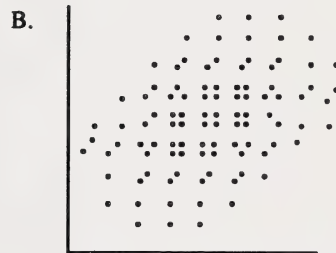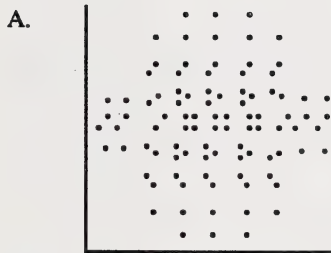
B. $y = 2\cos\left(\theta + \dfrac{\pi}{2}\right)$ $\qquad -2\pi \le \theta \le 2\pi$

C. $y = -2\sin\left(\theta - \dfrac{\pi}{2}\right)$ $\qquad -2\pi \le \theta \le 2\pi$

D. $y = -2\cos\left(\theta - \dfrac{\pi}{2}\right)$ $\qquad -2\pi \le \theta \le 2\pi$

9. On a certain cold day, the mean waiting time for a taxi was 40 minutes, with a standard deviation of 12 minutes. Of 50 callers, the number who received a taxi within 30 minutes was

    A.   10

    B.   20

    C.   30

    D.   40

10. Refrigerator manufacturers determine the mean life of their refrigerator motors to be 11 years, with a standard deviation of 4 years. If they guarantee their refrigerator motors for 3 years, the percentage of motors that will have to be replaced is

    A.   47.72%

    B.   0.47%

    C.   0.27%

    D.   2.28%

11. Which of the following graphs shows a strong positive correlation?

12. For a normal distribution, the mean (μ) is 15, and the standard deviation (σ) is 6. The probability to four decimal places of x for the interval $10.2 \leq x \leq 12.8$ is

    A. 0.4287

    B. 0.6436

    C. 0.1438

    D. 0.2664

13. Assuming the sequence $-7, -4, -1, 2, 5, 8, \ldots$ is arithmetic, the 27th term is

    A. 71

    B. 74

    C. 81

    D. 78

14. The sum of n terms in a geometric series is 171. If the first term is 1 and the common ratio is -2, then the number of terms in the series is

    A. 11

    B. 9

    C. 7

    D. 5

15. In an arithmetic series the sum of n terms is 788.5. The first and last terms in this series are 10 and 73 respectively. The number of terms in the series is

    A   12.

    B. 13

    C. 18

    D. 19

16. For the geometric series $3 + 3\sqrt{3} + 9 + \ldots$, the sum of the first seven terms is

   A. $120 + 39\sqrt{3}$

   B. $120 - 39\sqrt{3}$

   C. $240 + 39\sqrt{3}$

   D. $240 - 39\sqrt{3}$

17. Using sigma notation, the series
   $(2 \times 5) + (4 \times 7) + (6 \times 9) + \ldots + (12 \times 15)$ is

   A. $\displaystyle\sum_{k=1}^{6}(k)(k+3)$

   B. $\displaystyle\sum_{k=1}^{6}(2k)(2k+3)$

   C. $\displaystyle\sum_{k=0}^{4}(k+2)(k+5)$

   D. $\displaystyle\sum_{k=1}^{6}(k+2)(2k+3)$

18. If $2^x = 5$, then $\log_2(25)$ in terms of x is

   A. 10^x

   B. 3^{5x}

   C. x^5

   D. $2x$

19. $\dfrac{\log d}{h} + \dfrac{m \log t}{h}$ is equal to

     A. $\log \sqrt[h]{d \times mt}$

     B. $\log \sqrt[h]{\dfrac{t^m}{d}}$

     C. $\log \sqrt[h]{dt^m}$

     D. $\log dt^m - h$

20.   The value of x in the equation $2^x = 3^{x+1}$ is

     A. -2.709

     B. 3.8

     C. 0.602

     D. 0.48

21.   If $\log 4x - \log \dfrac{x}{2} = y$, then its exponential form is

     A. $10^y = 8$

     B. $8^y = 10$

     C. $y^8 = 10$

     D. $y^{10} = 8$

Use the information below to answer question 22.

$$N(t) = N_0 \left(2^{\frac{t}{40}} \right), \text{ where } N(t) = \text{final number of bacteria,}$$
$$N_0 = \text{initial number of bacteria,}$$
$$\text{and} \quad t = \text{time in minutes.}$$

22. The logarithmic expression for the time (t) it takes the number of bacteria to increase from 10 000 to 400 000 is

A. $\dfrac{\log_2 40}{40}$

B. $\log_2 \left(2^{\frac{1}{40}} \right)$

C. $\dfrac{40}{\log_2 40}$

D. $\dfrac{40 \log 40}{\log 2}$

23. The equation of a circle with centre (0, 0) and a y-intercept of -3 is

A. $x^2 + (y-3)^2 = 0$

B. $(x-3)^2 + y^2 = 0$

C. $x^2 + y^2 = 9$

D. $x^2 + (y-3)^2 = 9$

24. A circle with centre $(-2, 3)$ and tangent to the line $y = 0$ is defined by the equation

A. $(x+2)^2 + (y-3)^2 = 9$

B. $(x-2)^2 + (y+3)^2 = 9$

C. $(x+2)^2 + (y-3)^2 = 4$

D. $(x-2)^2 + (y+3)^2 = 4$

25.   If the equation of a parabola is $y^2 = -8x$, then the focus and directrix respectively are

   A. $F(2, 0)$, $x = 2$

   B. $F(2, 0)$, $x = -2$

   C. $F(-2, 0)$, $x = 2$

   D. $F(-2, 0)$, $x = -2$

26.   What is the equation of a parabola for which the vertex is (4, 2) and the focus is 3 units to the left of the vertex?

   A. $(x-4)^2 = -12(y-2)$

   B. $(y-2)^2 = 12(x-4)$

   C. $(y-2)^2 = -12(x-4)$

   D. $(x+4)^2 = 12(y+2)$

27.   The equation of a circle is $x^2 + y^2 - 4x + 6y - 3 = 0$.
       The centre and radius respectively of this circle are

   A. $(-2, 3)$,   16

   B. $(2, -3)$,   16

   C. $(-2, 3)$,   4

   D. $(2, -3)$,   4

28.   If the equation of the ellipse is $25x^2 + 16y^2 = 400$, then the length of the major axis is

   A.   4

   B.   5

   C.   8

   D.   10

29. The equation of the ellipse centred at the origin with a major axis of 18 units and one focus at $(0, -7)$ is

    A. $\dfrac{x^2}{81} + \dfrac{y^2}{32} = 1$

    B. $\dfrac{x^2}{81} - \dfrac{y^2}{32} = 1$

    C. $\dfrac{x^2}{32} + \dfrac{y^2}{81} = 1$

    D. $\dfrac{x^2}{81} + \dfrac{y^2}{49} = 1$

30. A point (x, y) moves so that the difference between its distances from $(0, 7)$ and $(0, -7)$ is always 10. One possible value of x when y equals 6 is

    A. $\dfrac{\sqrt{264}}{5}$

    B. $\sqrt{\dfrac{300}{24}}$

    C. $\dfrac{\sqrt{1464}}{5}$

    D. $\sqrt{\dfrac{1500}{24}}$

31. The length of the conjugate axis of a hyperbola whose centre is at the origin is 24, and the length of the transverse axis is 10. How far is the focus from the origin?

    A.   11

    B.   13

    C.   22

    D.   26

32. The equation of the hyperbola with centre at (0, 0), one focus at (0, 5), and one vertex at (0, – 2) is

A. $\dfrac{x^2}{21} - \dfrac{y^2}{4} = 1$

B. $\dfrac{y^2}{21} - \dfrac{x^2}{4} = 1$

C. $\dfrac{x^2}{4} - \dfrac{y^2}{21} = 1$

D. $\dfrac{y^2}{4} + \dfrac{x^2}{21} = 1$

33. The graph to the right is the graph of one of the functions below. The function is

A. $x^3 - x^2 - 5x - 3$

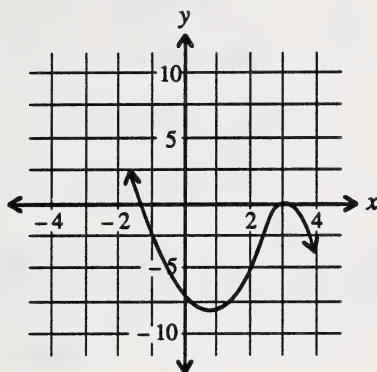B. $x^3 - x^2 - 4x - 6$
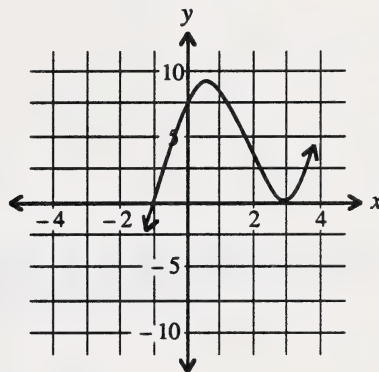
C. $x^3 + x^2 - 5x + 3$

D. $x^3 - 5x^2 + 3x + 9$

34. If $y = x^3 - 5x^2 + 3x + 9$, then the sketch that best represents the graph is
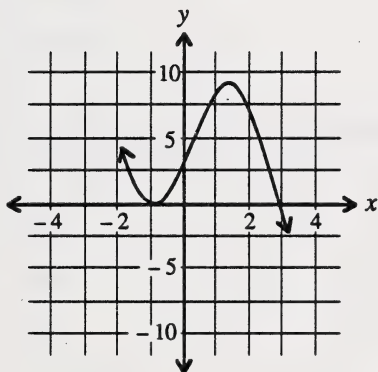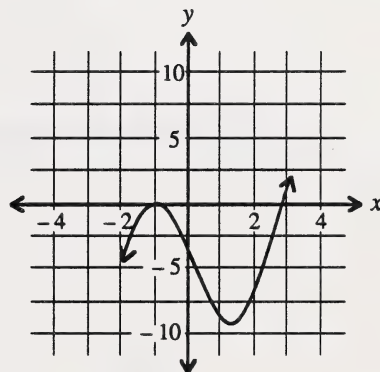
A.

B.

C.

D.

35. The coefficient of x^3 in a third-degree polynomial $P(x)$ is 2. If $P(2) = P(-2) = P(3) = 0$, then $P(x)$ is equal to

A. $2x^3 + 2x^2 + 8x - 24$

B. $2x^3 - 6x^2 - 8x + 24$

C. $2x^3 - 2x^2 - 8x - 24$

D. $2x^3 + 14x^2 + 8x - 24$

36. When $3x^3 - 4x^2 + 5x - 6$ is divided by $x - 1$, the quotient and remainder are

 A. $3x^2 - x + 4$ and -2

 B. $3x^2 - 7x + 12$ and -18

 C. $3x^2 - x + 6$ and 0

 D. $3x^2 + 7x + 12$ and -18

37. In how many ways can four different chemistry books and six different biology books be arranged on a shelf if the chemistry books are always kept together?

 A. 2880

 B. 17 280

 C. 120 960

 D. 967 680

38. In how many ways can eight people be seated at a round table if three of them must always sit together?

 A. 720

 B. 1140

 C. 4320

 D. 5040

39. How many different signals can be made with five different flags by raising 1, 2, 3 or 4 flags at a time on a flagpole? (No flags is not considered as a signal.)

 A. 320

 B. 205

 C. 30

 D. 26

40. A committee of four is to be selected from seven men and eight women. How many different committees can be formed if there must be at least one woman?

   A. 960

   B. 1130

   C. 1330

   D. 1560

41. There are five girls and three boys in a class. If a group of three students is chosen at random to form a committee, determine the probability that two of them are boys.

   A. $\dfrac{17}{56}$

   B. $\dfrac{15}{56}$

   C. $\dfrac{13}{56}$

   D. $\dfrac{11}{56}$

PART A: RESPONSE PAGE

_____	1.	_____	11.	_____	21.	_____	31.	_____	41.
_____	2.	_____	12.	_____	22.	_____	32.		
_____	3.	_____	13.	_____	23.	_____	33.		
_____	4.	_____	14.	_____	24.	_____	34.		
_____	5.	_____	15.	_____	25.	_____	35.		
_____	6.	_____	16.	_____	26.	_____	36.		
_____	7.	_____	17.	_____	27.	_____	37.		
_____	8.	_____	18.	_____	28.	_____	38.		
_____	9.	_____	19.	_____	29.	_____	39.		
_____	10.	_____	20.	_____	30.	_____	40.		

Name of Student _____   Student I.D. # _____

Name of School _____   Date _____

PART B: OPEN-ENDED SCANNABLE QUESTIONS

Record your answer next to each question by writing it in the corresponding answer field and by filling in one circle in EVERY column as illustrated. If you wish to change an answer, please erase your first answer completely.

Examples:

a.   If 10, x, 90 are consecutive terms of a geometric sequence, then a positive value of x to the nearest tenth is _____.

$$\frac{x}{10} = \frac{90}{x}$$

$$x^2 = 900$$

$$x = \pm 30$$

Record 030.0.

$$3, \ 9, \ 27, \ \ldots, \ 3^n, \ \ldots$$

b.   If $\cos \theta = 0.7821$ where $0° \leq \theta \leq 90°$, then the measure of θ correct to the nearest tenth of a degree is _____ .

$\theta = 38.546\ 746\ 41$
Record 038.5.

Name of Student _____   Student I.D. # _____

Name of School _____   Date _____

Value

(2)   1.   It is assumed that the science test scores are normally distributed with $\mu = 80$ and $\sigma = 25$.
              The probability, to the nearest tenth, that a student's score will be more than 87 is

              _____ .

(2)   2.   In a geometric sequence, the first term is 10 and the fourth term is 80. The seventh term,
              correct to the nearest tenth, is _____ .

Name of Student _____   Student I.D. # _____

Name of School _____   Date_____

Value

(2)

3. If the remainder is 43 when $2x^3-3x^2+4x+m$ is divided by $x-3$, m, correct to the nearest tenth, is _____ .

			.	
⓪	⓪	⓪	.	⓪
①	①	①	.	①
②	②	②	.	②
③	③	③	.	③
④	④	④	.	④
⑤	⑤	⑤	.	⑤
⑥	⑥	⑥	.	⑥
⑦	⑦	⑦	.	⑦
⑧	⑧	⑧	.	⑧
⑨	⑨	⑨	.	⑨

(2)

4. If $\log_{32}(x) = -\frac{1}{5}$, then the value of x correct to the nearest tenth is _____.

			.	
⓪	⓪	⓪	.	⓪
①	①	①	.	①
②	②	②	.	②
③	③	③	.	③
④	④	④	.	④
⑤	⑤	⑤	.	⑤
⑥	⑥	⑥	.	⑥
⑦	⑦	⑦	.	⑦
⑧	⑧	⑧	.	⑧
⑨	⑨	⑨	.	⑨

Name of Student _____   Student I.D. # _____

Name of School _____   Date _____

Value

(2)   5.   A pendulum 50 cm long swings through an angle of 36°. What is the length of the arc, correct to the nearest tenth of a centimetre, through which a spider swings if it clings to the pendulum 45 cm from its top?

Value

PART C: WRITTEN RESPONSE

Be sure to show all your work. Part marks will be given for partially correct solutions.

(4)   1.   In a geometric series, the sum of the first five terms is -7320 and the sum of the first six terms is 21 840. The fourth term is 3240. Calculate S_9 for the series.

Value

(4)   2.   Express the polynomial $3x^4 - 5x^3 - 5x^2 + 5x + 2$ as a product of first-degree factors.

Name of Student _____   Student I.D. # _____

Name of School _____   Date_____

Value

(4)   3.   Determine the sixth term in the expansion of $(2x-3y)^9$.
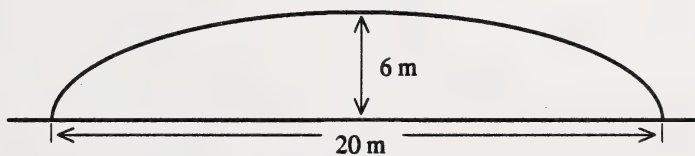
(4)   4.   The roof of an arena is in the shape of one half of an ellipse. Use the dimensions shown to find the equation of the ellipse.

Value

④   5.   An I.Q. test was given to a group of students. The result follows a normal distribution with a mean of 120 and a standard deviation of 15. Between what two values on either side of the mean will 10.9% of the scores be found?

Name of Student _____   Student I.D. # _____

Name of School _____   Date _____

Value

(4)

6. Draw the line of best fit for the following scatterplot using the median fit method.

Value

④ 7. An environmental group conducted a survey of forty people to find out if they thought that atmospheric pollution was on the increase. Twenty four persons responded yes to the question. Use the chart below to answer the questions that follow.

90% Box and Whisker Plots from Samples of Size 40

Proportion of Yeses in Sample

Percentage of Yeses in Population

Number of Yeses in Sample
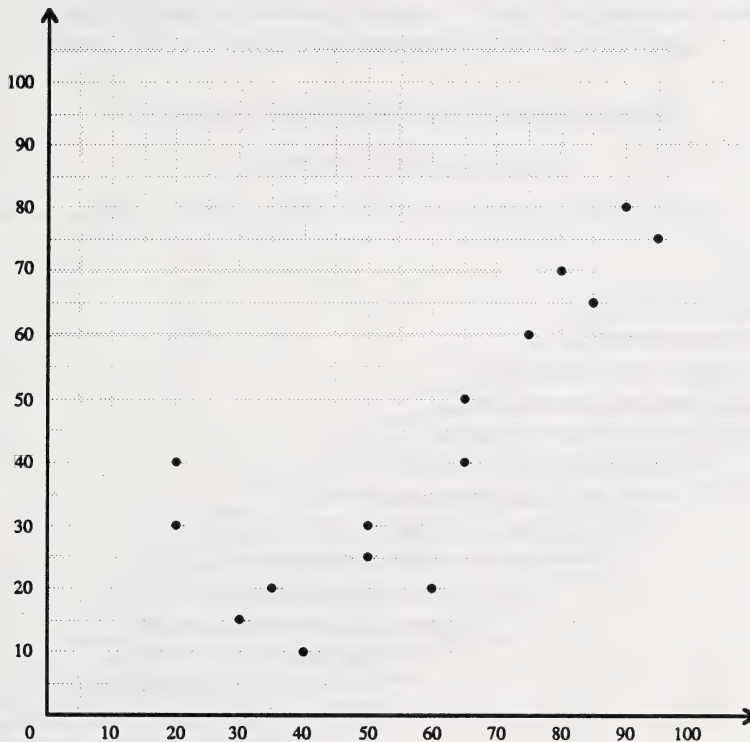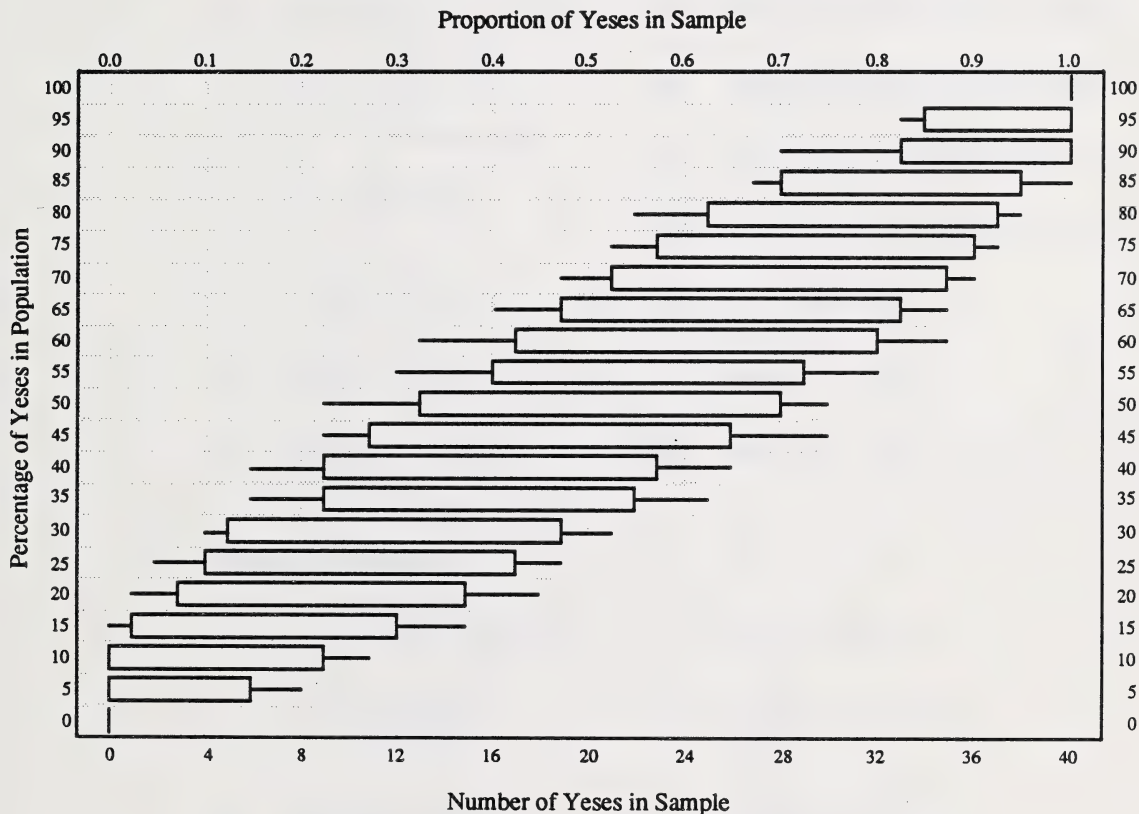
a. What is the confidence interval?

b. Explain what is meant by the confidence interval in a.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

MATHEMATICS 30
FORMULA SHEET

A. Trigonometry

1. $\pi = 3.14159$

2. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

3. $a^2 = b^2 + c^2 - 2bc \cos A$

4. $\sin^2 A + \cos^2 A = 1$

5. $1 + \tan^2 A = \sec^2 A$

6. $1 + \cot^2 A = \csc^2 A$

7. $\sin\left(\dfrac{\pi}{2} - \theta\right) = \cos \theta$

8. $\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin \theta$

9. $\sin (A + B) = \sin A \cos B + \cos A \sin B$

10. $\sin (A - B) = \sin A \cos B - \cos A \sin B$

11. $\cos (A + B) = \cos A \cos B - \sin A \sin B$

12. $\cos (A - B) = \cos A \cos B + \sin A \sin B$

13. $\sin (-\theta) = -\sin \theta$

14. $\cos (-\theta) = \cos \theta$

15. $\tan (-\theta) = -\tan \theta$

B. Quadratic Relations

1. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

3. $M\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

4. $(x - h)^2 + (y - k)^2 = r^2$

5. $x^2 + y^2 + Dx + Ey + F = 0$

6. $(y - k)^2 = 4p(x - h)$

7. $(x - h)^2 = 4p(y - k)$

8. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, a^2 = b^2 + c^2$

9. $\dfrac{y^2}{a^2} + \dfrac{x^2}{b^2} = 1, a^2 = b^2 + c^2$

10. $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1, c^2 = a^2 + b^2$

11. $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1, c^2 = a^2 + b^2$

C.   Sequences, Series, and Limits

1. $t_n = a + (n-1)d$

2. $S_n = \dfrac{n(a + t_n)}{2}$

3. $S_n = \dfrac{n[2a + (n-1)d]}{2}$

4. $A = P(1+i)^n$

5. $t_n = ar^{n-1}$

6. $S_n = \dfrac{a(r^n - 1)}{r - 1}$

7. $S_n = \dfrac{rt_n - a}{r - 1}$
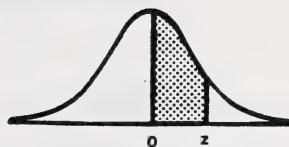
8. $S = \dfrac{a}{1-r}, -1 < r < 1$

D.   Statistics

1. $\mu = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$

2. $\sigma = \sqrt{\dfrac{(x_1 - \mu)^2 + \cdots + (x_n - \mu)^2}{n}}$

3. $z = \dfrac{x - \mu}{\sigma}$

**Areas
under the
Standard
Normal Curve**

z	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

Teacher Questionnaire

Course name & number _____   Teacher's name _____

Teacher's area of expertise _____   School name _____

Date _____

Design

1. The modules follows a definite systematic design. Did you find it easy to follow?

   Yes   No   If no, explain.

   _____

   _____

2. Did your observations reveal that the students found the design easy to follow?

   Yes   No   If no, explain.

   _____

   _____

3. Did you find the Learning Facilitator's Manual helpful?

   Yes   No   If no, explain.

   _____

   _____

4. Part of the design involves stating the objectives in student terms. Do you feel this helped the students understand what they were going to learn?

   Yes   No   If no, explain.

   _____

   _____

5. The activities include self-assessment questions. These questions are to help clarify and reinforce the instructional materials. Did students use the answers effectively?

Yes   No   If no, explain.

_____

_____

6. Did the two streams of Follow-up Activites (Extra-help and Enrichment) prove to be helpful?

Yes   No   If no, explain.

_____

_____

7. Were students motivated to try these Follow-up Activities on their own?

Yes   No   If no, give details.

_____

_____

8. Were the assignments clear?

Yes   No   If no, give details.

_____

_____

9. Were the assignments appropriate?

Yes   No   If no, give details.

_____

_____

10. Did you fax assignments?

Yes   No

11. If you did fax, did you get satisfactory results from using this procedure?

Yes   No   If no, give details.

_____

_____

_____

Instruction

1. Did you find the instruction clear?

Yes   No   If no, give details.

_____

_____

2. Did your observations reveal that the students found the instruction interesting?

Yes   No   If no, give details.

_____

_____

3. Did you find the instruction adequate?

Yes   No   If no, give details.

_____

_____

4. Was the reading level appropriate?

Yes   No   If no, give details.

_____

_____

5. Was the work load adequate?

Yes   No   If no, give details.

_____

_____

6. Was the content accurate and current?

Yes   No   If no, give details.

_____

_____

7. Did the content flow consistently and logically?

Yes   No   If no, give details.

_____

_____

8. Was the transition between booklets smooth?

Yes   No   If no, give details.

_____

_____

9. Was the transition between print and media smooth?

   Yes   No   If no, give details.

   _____

   _____

Additional Comments

   _____

   _____

   _____

   _____

   _____

   _____

   _____

   _____

When you have completed this questionnaire, please mail it to the following address.

        Design Department
        Alberta Distance Learning Centre
        Box 4000
        Barrhead, Alberta
        T0G 2P0